A PROPOSAL FOR THE CONTROL OF PROPELLANT UTILIZATION IN A LIQUID BIPROPELLANT ROCKET ENGINE

THOMAS M. KASTNER
AND CARLOS D. McCULLOUGH

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943-5002

DU
NA
MO

A PROPOSAL FOR THE CONTROL OF PROPELLANT UTILIZATION IN A LIQUID BIPROPELLANT ROCKET ENGINE

by

THOMAS M. KASTNER

and

CARLOS D. McCULLOUGH

MAY 1957

University of Michigan

K 145
c.1

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943-5002

TABLE OF CONTENTS

35888

SUMMARY

The purpose of this study was the examination of the general problem of propellant utilization in large liquid bipropellant rocket engines, and consideration of the feasibility of effecting desired utilization with a particular method of control.

The proposed control consists of a closed loop system whereby the operating mixture ratio is maintained at a specified level which in turn is assumed equal to the loading ratio of propellants in the vehicle tanks. The necessary control over propellant flow to the thrust chamber is accomplished through the use of differentially operated control valves located in bypass lines about each feed pump.

Electronic differential analyzer simulation of the application of such a control system to a typical large rocket engine indicates that in the low frequency range the overall system dynamic behavior is essentially that of the controller. Controller action is limited to low frequencies in order to avoid the possibility of interaction of flow modulation and combustion delay introducing unstable thrust chamber operation.

A comparison of the use of a bypass line about each feed pump as opposed to a single bypass line about one feed pump indicates that the former is preferable when corrections in mixture ratio are expected to exceed 2.5 per cent and it is desirable to maintain the engine performance within 1 per cent of design thrust. A dual line control is capable of meeting this thrust performance requirement for corrections in mixture ratio of up to 10 per cent.

Practical considerations of available component limitations indicate that

a dual line bypass control system is capable of response times in the order of .07 to .11 seconds and an expected accuracy of .5 per cent.

SYMBOLS

A	area
F	thrust, pounds
I_{sp}	specific impulse, seconds
K	constant
L^*	characteristic length, feet
N	pump speed
P	pressure, pound per square inch
Q	flow rate, cubic feet per second
R	loading ratio, pounds of oxidizer per pound of fuel in rocket tanks.
R'	universal gas constant
T	absolute temperature, degrees Rankine
V	volume, cubic feet
W	weight, pounds
Y	transfer function
$\mathcal{M}$	molecular weight, pounds per mole
$\mathcal{T}$	actuator time constant, seconds
c^*	characteristic velocity, feet per second
g	gravitational constant, 32. 2 feet per second per second.
h	altitude, feet
k	constant
l	length, feet
m	mass, slugs

p	$\frac{d}{dt}$
r	mixture ratio
s	$\frac{d}{d\tau}$
t_b	time of burn out, seconds
t_c	combustion delay time, seconds
t_s	chamber stay time, seconds
Δ	increment of change
γ	ratio of specific heats
ϵ	error signal
ρ	density, slugs per cubic foot
θ	valve position
ω	angular frequency, radians per second
ζ	damping factor

Subscripts

b	bypass
c	chamber
f	fuel
i	injector or initial value
o	oxidizer

A PROPOSAL FOR THE CONTROL OF

PROPELLANT UTILIZATION IN A LIQUID

BIPROPELLANT ROCKET ENGINE

INTRODUCTION

The performance of a rocket propelled vehicle is greatly influenced by its mass at any given instant of time. The advent of large rockets, where structural weight is but a fraction of the propellant weight, coupled with strict performance tolerances requires that the utilization of the propellants be subject to extremely close control.

Early attempts at establishing this control consisted primarily of calibration of the engine by ground testing. This method, while able to compensate for various component tolerances, is unable to correct for any in-flight disturbances such as acceleration effects or large changes in ambient conditions.

The purpose of this study was to investigate the use of a closed loop control system in order to maintain the propellant utilization at some desired level. The proposed control system is based on the use of bypass flow control around each propellant feed pump. The application of the proposed control system to a typical rocket propelled vehicle was studied using, in part, analog computer simulation.

Rocket engine simulation has previously been treated at length by Gore and Carroll, (Ref. 1.); B. N. Smith,(Ref. 2.); and Sanders, Novich, and Hart, (Ref. 3.). The methods used in this study represent a modification of those presented by Smith. The theory used to describe rocket engine performance

as a function of propellant flow and mixture ratio represents a simplification of similar considerations by Crocco, (Ref. 4.).

This study was undertaken by T. M. Kastner and C. D. McCullough as a joint thesis project to satisfy, in part, the requirements for a Master of Science Degree in Aeronautical Engineering at the University of Michigan, Ann Arbor, Michigan, September 1956 - June 1957.

THE NATURE OF THE PROBLEM

OBJECTIVES

The nature of any propellant utilization system depends on the objective of its inclusion as part of the rocket engine. This objective, while not singular in concept, is determined in the main by the performance requirements of the rocket vehicle. The overall objective may be the attainment of one or any combination of the following requirements:

1. Maintaining engine operation within certain specified limits.
2. Continuous control of vehicle mass.
3. Obtaining the maximum energy transference from the propellant on board. This will call for total propellant utilization during the flight. In the case of a bipropellant rocket this may be interpreted to mean that both fuel and oxidizer are exhausted simultaneously with engine burn out.
4. The future propellant requirements on a given flight may establish a need for varying degrees of propellant utilization.

It might seem that these requirements can equally well be met by the proper choice of design parameters in the basic engine system. Unfortunately

the absolute achievement of any performance level is an impossibility. In reality there are only degrees of achievement within certain error limits. In order to better understand the problem of propellant utilization it is necessary to examine the nature of these errors and their effect upon engine and vehicle performance.

CAUSES FOR DEVIATIONS FROM PERFECT PROPELLANT UTILIZATION

In order to effect the control of propellant utilization it is necessary to consider the causes for the pertubations from the original engine design conditions. Improper propellant utilization is most evident in a shift from the design mixture ratio where operation at other than the design mixture ratio may be attributed to the following:

1. Variation in propellant density. The basic engine design assumes specific values for the densities of the oxidizer and fuel. These densities are based upon assumed flow temperatures. If the propellant temperatures differ from the assumed values there may be a change in the operating mixture ratio. This can be shown by considering the flow at the thrust chamber injector where

$$\dot{m}_o = K_1 A_{io} \sqrt{2(P_{io} - P_c)\rho_o}$$

and

$$\dot{m}_f = K_2 A_{if} \sqrt{2(P_{if} - P_c)\rho_f}$$

The mixture ratio may be expressed as

$$r = K\sqrt{\frac{\rho_o}{\rho_f}}$$

or in differential form

$$dr = \frac{r}{2}\left(\frac{d\rho_o}{\rho_o} - \frac{d\rho_f}{\rho_f}\right)$$

Considering some variation in propellant temperatures ΔT_o and ΔT_f , with

$$d\rho = \frac{d\rho}{dT}\, dT$$

the change in r resulting from the temperature induced density shift may be expressed as

$$dr = \frac{r}{2}\left[\frac{\left(\frac{\partial\rho}{\partial T}\right)_o dT_o}{\rho_o} - \frac{\left(\frac{\partial\rho}{\partial T}\right)_f dT_f}{\rho_f}\right]$$

If an identical temperature shift is assumed for each propellant

$$\frac{dr}{dT} = \frac{r}{2}\left[\frac{\left(\frac{\partial\rho}{\partial T}\right)_o}{\rho_o} - \frac{\left(\frac{\partial\rho}{\partial T}\right)_f}{\rho_f}\right]$$

or in terms of some perturbation from the design value $\bar{T}$,

$$\Delta r = \frac{\bar{r}}{2}\left[\frac{\left(\frac{\partial\rho}{\partial T}\right)_o}{\bar{\rho}_o} - \frac{\left(\frac{\partial\rho}{\partial T}\right)_f}{\bar{\rho}_f}\right]$$

From the above it is easily seen that for any condition other than

$$\frac{\left(\frac{\partial\rho}{\partial T}\right)_o}{\left(\frac{\partial\rho}{\partial T}\right)_f} = \frac{\bar{\rho}_o}{\bar{\rho}_f}$$

a shift in mixture ratio will result from a change in propellant injection temperatures. This effect is shown in Figure 1 for several different propellant combinations. Note that only in the case of nitric acid and aniline is this condition approached. It may easily be shown that for a LOX-ETHYLAC combination

$$\frac{dr}{dT} = .353 \times 10^{-3} / {}^\circ R$$

Here a 50° temperature shift will result in a 1.3% change in r. It is surmised that some statistical variation in component densities may exist depending mainly upon supply conditions.

2. Variations in initial loading ratio R. In general the initial loading of the rocket will vary statistically with the probability of any specific loading ratio given as

$$P(R \pm \Delta R) = \int_{R-\Delta R}^{R+\Delta R} g(R)\,dR$$

where R is some increment of measure and g(R) is the distribution function for R. The nature of g(R) will depend on the accuracy with which the initial propellant loading can be determined. This particular perturbation would seem to lie outside the realm of total correction by propellant utilization system. However, if the nature of g(R) is known it is possible to reduce the likelihood of any resulting error to a minimum by controlling the operating mixture ratio such that there is an equal probability of having an excess of either fuel or oxidizer at burn out. This consideration is dealt with at greater length by Brousseau in Ref. 5.

3. A similar statistical perturbation will result from production tolerances in the system components. Here the probability of any operating mixture ratio will be of the form

$$P(r \pm \Delta r) = \int_{r-\Delta r}^{r+\Delta r} g(r)\,dr$$

where g(r) is the distribution function of r. It is possible to correct this perturbation within the limits of accuracy of measurement of the instanteous of operating mixture ratio and the capability of the control system. Indeed, one principal advantage of any such control system is the fact that its use might allow relaxation of production tolerances of certain engine system components and thus enhance over-all production.

4. Accelerations experienced by the rocket during its flight. One effect of acceleration is such as to vary the hydrostatic contribution to injection pressure of each propellant component. Here the resulting change in mixture ratio can be shown to be approximately

$$\Delta r = \frac{\bar{r}}{2}\left(\frac{\Delta P_{io}}{\bar{P}_{io}} - \frac{\Delta P_{if}}{\bar{P}_{if}}\right)$$

Most of this effect is due to variations in the inlet pressure of the feed pumps due to the change of hydrostatic head in the propellant tanks. The effect may be compensated by adjusting the tankage and line size in such a way to insure that

$$\frac{\Delta P_{\Lambda o}}{\Delta P_{\Lambda f}} \doteq \frac{\bar{P}_{\Lambda o}}{\bar{P}_{\Lambda f}}$$

Reichel in Ref. 6. considers the advantages of a concentric tank arrangement towards this end. Other possible schemes include a variable tank pressurization system to insure constant pump suction head or use of special pump design and compensation.

EFFECTS OF IMPROPER UTILIZATION

Justification for the use of a propellant utilization control system may be best effected by considering the results of improper utilization. If a rocket vehicle, such as that of Appendix I, is initially loaded with propellants apportioned so that engine operation at the optimum mixture ratio will result in complete burning of all propellants during the flight any improper utilization, as represented by operation at other then the designed mixture ratio, will result in the following:

1. There will be residual propellant on board at burn out which will act to decrease the effective mass ratio of the rocket and thus its performance.

2. The burning time will be decreased.

3. There will be a deviation from the design specific impulse which will change the performance.

4. The shift in mixture ratio, if large, may cause temperature limits of the engine to be exceeded.

5. Improper propellant utilization may result in low frequency combustion instability as evidenced in the experimental work Barres and Moutet,(Ref. 7).

6. Any variation in the total mass flow rate of the propellant will affect the engine thrust. This effect is generally of minor importance and can either increase or decrease performance.

Appendix II demonstrates some of the more marked effects of improper propellant utilization on a typical large rocket. Other examples are given by Rosen,(Ref. 8.) and Reichel,(Ref. 6.). In Ref. 8, Rosen vividly illustrates the performance problem by calculating the expected performance of three theoretical satellite proposals with an allowance of .025 of the initial propellant weight remaining on board at burn out. (.025 is an average figure for the initial Viking series.) The corrected orbits fell far short of the ideal calculations based upon perfect utilization and in one instance orbital velocity was not attained.

PROPOSED CONTROL SYSTEM

DESCRIPTION

In conducting this study it was necessary to assume requirements that seemed typical to present applications of the large rocket engine. The essential objective of the control system proposed is the ability to effectively control the propellant consumption throughout the engine burning time in such a manner as to insure complete utilization of all propellant components at burn out. Other considerations are that engine performance with respect to thrust and chamber pressure should be essentially constant and that the system be compatible with stable engine operation throughout its control range.

Any such system must of course be restricted in its application to installations in which the added weight and complexity of the system do not offset the advantages gained by its employment.

There are several means by which control of propellant utilization may be

effected. Basically these consist of first, some means of measuring any departure from the desired utilization criteria and second, some means of compensation for the detected error. The measurement problem may be considered as one of determining the propellant on board and of determining the instanteous operating mixture ratio. Use of flow rate measurement methods seems to offer the most hope of accomplishment with the least error. Compensation for improper utilization will generally require that the propellant in excess be disposed of in order to avoid the performance limitations imposed by its addition to the rocket weight. This may be accomplished by jettisoning or by burning in the thrust chamber. Jettisoning of fuel from a rocket with the engine operating is generally not desirable. It means a waste of fuel energy and, it does not correct any deviations in operating mixture ratio. It does have the advantage of not disturbing the thrust chamber flow. The second method is basically one of maintaining the mixture ratio at a specified level by modulation of the flow rates. A reasonably rapid response in such a system will avoid any variation in desired utilization provided the specified operating mixture ratio and the loading ratio of the rocket are identical. This assumes a knowledge of the loading ratio at time of lift off through launch pad instrumentation and a corresponding setting of the operating mixture ratio.

The engine and control system as shown schematically in Figure 2. In this system the feed pump speeds are assumed to remain constant. The flow rate in each propellant line at the injector is varied by using a controlled bleed or bypass from the injector inlet to the pump suction. Mass flows are obtained from the volumetric flow rate output of vane-type flow meters which

are calibrated against propellant temperature,

$$\dot{m} = \rho Q$$

where

$$\rho = f(T)$$

The method of controlling bypass flow is shown in Figure 3. Here the operating mixture ratio is compared with the desired ratio, R, and the difference or error signal is used to position the flow valves. The valve motions of the control bypass lines are opposite in sense. This differential action is made a function of the error signal by inversion of the signal supplied to one valve actuator. Another possible solution is the use of a single actuator with differential gearing of the valve action.

In determining the nature of the control system components several assumptions regarding the effects of the control system upon the response of the engine are made.

1. Controlled flow rate perturbations are to be low enough in frequency so that any phase lag occuring due to combustion delay time will be negligible. The delay time is assumed to be approximately 0.002 seconds, (Ref. 9.). Therefore, it is required there be no control perturbations with a frequency much greater than 50 radians per second.

2. It is desired that the thrust chamber pressure not vary more than 2% from the design value.

3. The control system should be capable of correcting for possible deviations of 10% in operating mixture ratio.

4. The rate of change of mixture ratio correction is to be limited such that there is no more than 1% variation in mixture ratio between injector flow and chamber exit flow as a result of control action. This requires an error signal limitation of 10% $\bar{r}$. Also, assuming that the maximum

corrective action is demanded in the form of a step response, if the rate of change of mixture ratio is held so that there is no more than a 10% response to the error signal within a period equal to the chamber stay time the rate limitation will not be exceeded. This allows the assumption of uniform conditions throughout the chamber.

The limitations regarding frequency and rate of flow modification as a result of controller action are necessitated by stability considerations of the basic system composed of thrust chamber and feed lines. Studies of the low frequency dynamics of rocket engines show the dependence of stability upon the phase lag between injector flow and chamber pressure. (See References 7, 9, 10, 11, and 12). The contribution of combustion delay time to this phase lag is of primary importance in stability considerations. In as much as the behavior of the delay time under conditions of varying mixture ratio as well as overall mass flow is not yet fully understood it was considered desirable to restrict the control flow perturbations such that the phase lag due to any delay time would be less than one tenth radian.

SYSTEM EQUATIONS

Derivation of the equations describing the system is based on linearization about the design operating conditions. The following assumptions are made.

1. System components may be described by lumped parameters.
2. Suction heads for both pumps are constant.
3. System components are inelastic.

The equations describing the system are shown in Figure 4.

Pumps: The feed pumps are centrifugal flow pumps with typical head versus flow characteristics. Stepanoff, (Ref. 13) gives the following analytical expression for this type pump characteristic:

$$P_D - P_S = AN^2 + BQN + CQ^2$$

Given constant N and P_S the pump behavior may be expressed

$$\Delta P_D = \frac{\partial P_D}{\partial Q} \Delta Q = K_p \Delta Q$$

Similar characteristics were assumed for both fuel and oxidizer pumps.

Main Propellant Line: For small perturbations in flow the effect of line friction may be neglected with the only pressure drop due to the flow inertia. Thus

$$P_D - P_i = \ddot{m} \frac{l}{A} = \rho \frac{l}{A} (\dot{Q}_i + Q_b)$$

or

$$\Delta P_D - \Delta P_i = K_L (\Delta \dot{Q}_i + \Delta \dot{Q}_b)$$

Injector: Injector flow is given by the equation for orfice flow as

$$Q_i = C_{D_i} A_i \sqrt{\frac{2(P_i - P_c)}{\rho}}$$

or in linearized form as

$$\Delta P_i - \Delta P_c = \frac{2(P_i - P_c)}{Q_i} \Delta Q_i = K_i \Delta Q_i$$

Flow Sensing: The problem of accurate flow sensing, or measurement, is one of great importance in effecting any satisfactory propellant utilization control system. References 13, 14, and 15, describe several methods of approach. The dynamics of a vane type sensing instrument are considered in detail by Grey in Reference 14. Time constants of, from .5 to 3 milliseconds have been determined for such instruments. Therefore, the dynamics of the

flow sensors are assumed negligible with respect to the overall system dynamics.

Bypass Line: With similar assumptions as those for the main line

$$\Delta P_{s'} = \rho\left(\frac{l}{A}\right)_b \Delta Q_b = K_{lb}\,\Delta Q_b$$

Bypass Valve: In terms of valve position and flow the pressure drop across the valves is expressed as

$$P_i - P_{s'} = K\left(\frac{Q_b}{\theta}\right)^2$$

where θ represents valve position. In perturbation form

$$\Delta P_i - \Delta P_{s'} = \frac{2(P_i - P_{s'})}{Q_b}\Delta Q_b - \frac{2(P_i - P_{s'})}{\theta}\Delta\theta = K_b\,\Delta Q_b - K_\theta\,\Delta\theta$$

Thrust Chamber: The linearized equation representing the combustion chamber is developed at length in Appendix III and is given as

$$(1+S)\Delta P_c = K_{RO}\,\Delta Q_{io} + K_{RF}\,\Delta Q_{if} + K_{c*}(1+2S)\Delta r$$

Mixture Ratio: The mixture ratio expressed in linearized form is

$$\Delta r = \frac{r}{Q_{io}}\Delta Q_{io} - \frac{r}{Q_{if}}\Delta Q_{if}$$

Application of these equations to both propellant lines and the elimination of the pressure terms except for P_c results in the following expression for the system:

$$\begin{bmatrix} (1+s) & -K_{RO} & 0 & -K_{RF} & 0 & -K_{C^*}(1+2s) \\ 1 & K_{Lo} & -(K_b+K'_{Lo}s)_o & 0 & 0 & 0 \\ 1 & 0 & 0 & K_{if} & -(K_b+K'_{L}s)_f & 0 \\ 1 & (K_i-K_p+K'_L s)_o & -(K_p-K'_L s)_o & 0 & 0 & 0 \\ 1 & 0 & 0 & (K_i-K_p+K'_L s)_f & -(K_p-K'_L s)_f & 0 \\ 0 & 1/Q_{io} & 0 & -1/Q_{if} & 0 & -1/r \end{bmatrix} \begin{bmatrix} \Delta P_c \\ \Delta Q_{io} \\ \Delta Q_{if} \\ \Delta Q_{to} \\ \Delta Q_{tf} \\ \Delta r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -K\Delta\theta^* \\ -K\Delta\theta^* \\ 0 \\ 0 \end{bmatrix}$$

The constants K'_L etc. refer to a change of time variable $\tau = \frac{t}{t_s}$ to correspond to the time scaling of the thrust chamber equation.

$$K'_L = \frac{K_L}{t_s}, \quad \text{etc.}$$

While it is possible to use wholly analytical means to solve for the dynamics of the system, the solution of the determinants involved is laborious. Therefore, the use of analog computer simulation was chosen. It was considered desirable to check the solution for operation at design mixture ratio using the matrix equations to determine an equivalent transfer function representation of the mixture ratio response to a change in control valve position. This transfer function is of the general form

$$Y_{\Delta r/\Delta\theta}(s) = -K\frac{(s+\alpha_1)(s+\alpha_2)(s^2+\alpha_3 s+\alpha_4)}{(s+\beta_1)(s^2+\beta_2 s+\beta_3)(s^2+\beta_4 s+\beta_5)}$$

A Bode plot of the frequency response for the ideal case of r = 1.37 is shown in Figure 5. It is of interest to note that over the range of frequencies for which the control system is desired to act the transfer function is effectively constant and the phase shift is small.

---

* For dual valve control, $\Delta\theta = \Delta\theta_o = -\Delta\theta_f$

The control valve proposed is a gate type valve with an electric servo drive. A schematic of the valve is shown in Figure 6. A triangular orifice was chosen to insure constant discharge coefficient over the range of valve operation. Denoting the valve position for design flow condition, r = 1.37, as $\bar{\theta}$, valve positioning by controller action is represented in terms of non-dimensional valve movement $\frac{\Delta\theta}{\bar{\theta}}$. In order to obtain linearity between valve orifice area and actuator position, it is assumed that a nonlinear linkage exists such that

$$\frac{\Delta A}{\bar{A}} = \frac{\Delta\theta}{\bar{\theta}}$$

The over all valve and actuator motion may then be expressed as

$$\frac{\Delta\theta}{\bar{\theta}} \quad \frac{K_v}{p(Tp+1)}\,\epsilon_r$$

Here T represents the ratio of valve and actuator inertia to damping. K_v is the controller gain, and ϵ_r is the error in operating mixture ratio. In perturbation form

$$\epsilon_r = \Delta R - \Delta r$$

Scaled to the same time, $\tau = \frac{t}{t_s}$, as the previous equations,

$$\frac{\Delta\theta}{\theta} = \frac{K_{v\tau}}{S(T_\tau S+1)}\,\epsilon_r$$

where

$$K_{v\tau} = K_v t_s$$
$$T_\tau = \frac{T}{t_s}$$

When dual valve control with differential valve action is used

$$\frac{\Delta\theta}{\bar{\theta}} = \left(\frac{\Delta\theta}{\bar{\theta}}\right)_o = \left(\frac{\Delta\theta}{\bar{\theta}}\right)_f$$

SIMULATION

The computer simulation of the control system operation consisted of several phases, namely:

1. Study of the open loop dynamics of the basic system less controller over the expected range of operation,

$$.9 \leq \frac{r}{\bar{r}} \leq 1.1$$

2. Observation of the variation in equilibrium conditions as a result of control valve operation.

3. Observation of the closed loop response of the system for a step input of desired mixture ratio.

The computer circuit used to simulate the basic system is shown schematically in Figure 7. The basis for the computer circuit is the previously developed set of linear perturbation equations. The value of all constants except K_θ, K_b, and K_{c*} remain essentially constant over the range of flow conditions considered and were thus allowed to remain as calculated for operation at ideal design conditions. The values of K_θ, K_b, and K_{c*} were varied according to the equilibrium conditions about which the perturbations were assumed.

In the first phase a sinusoidal perturbation in $\frac{\Delta\theta}{\bar{\theta}}$ was applied to obtain the frequency response of the feed lines and thrust chamber. This was done about equilibrium conditions for r = 1.37, 1.50 and 1.26 which represented the optimum and extremes of operating limits.

The second phase consisted of the determination of equilibrium conditions

existing over the proposed range of control action. This was done by a summation of perturbation effects. Starting at the design condition, successive increments of $\Delta r = .02$ were used as a basis of calculation. This procedure was done for the following three modes of operation:

1. Bypass control in each propellant line
2. Bypass control in the fuel line only
3. Bypass control in the oxidizer line only

The final phase consisted of selecting a controller function which would give optimum response within the flow rate limitations dictated by the initial assumptions. This was done in terms of the system response to a step input for assumed values of valve actuator time constant T, of .01 and .02 sec. In addition, the frequency response of the closed loop system was obtained in order to observe the attenuation in flow variation with error signal frequency. The computer representation of the valve actuator and controller is shown in Figure 8.

RESULTS AND DISCUSSION

The results of the computer simulation for the first phase are shown in Figure 9. There was no attenuation in response for frequencies less than 180 radians per second. At this point the attenuation characteristics exhibited a -6 db. per octave roll off with no appreciable peaking at the break point. This attenuation characteristic was practically identical for both the extreme and optimum mixture ratio conditions. The accompanying phase lag of the mixture ratio response is small and approximately linear over the desired

frequency range, $\omega \leq 50$. Also shown in Figure 9 is the response predicted by analytical calculations for $r = 1.37$. Lack of complete correlation may be ascribed to measurement inaccuracies and simulation simplifications. These results lead to the initial conclusion that the basic engine design is stable over the desired range of operation, as predicted by analytical considerations. The desired control operation frequencies lie well below the break frequency of the line-chamber response and the phase lag is almost negligible in this region. From this it is evident that within the frequency range of 0 to 50 radians per second the response of the control system will be governed essentially by the response of the controller and valve actuator.

The results of the second phase of the computer simulation are shown in Figures 10, 11, and 12. Of primary interest is a comparison of the results for dual valve control with those for single valve control. As might be expected dual valve control offers a wider corrective range in mixture ratio for a given range of valve positions. In addition it should be noted that the dual valve control gain as represented by the slope of the mixture ratio versus valve position curve is not only larger but also more nearly constant for all valve positions than is true for single valve operation. In single valve operation there is a tendency for the gain to drop as the valve is opened. With dual valve control this effect is balance by the differential valve action. Observing the chamber pressure for each type of control it is seen that for dual valve control there is a gradual pressure drop for corrective action in either direction which is at most only five psi. at the extreme limits of the valve position. The control system requirements call for a chamber pressure variation limit of 2%. The variation

for dual valve control is well within this limit. Original system concepts had included a chamber pressure feed back in which the dual valves would be actuated in unison as well as differentially to compensate for any large variations in chamber pressure. In view of the small variations occuring within any feedback provisions it is felt that over the correction range chosen this refinement is not necessary. The differential action of the valves keeps the mass flow into the chamber nearly constant and most of the pressure drop experienced is the effect of shifting the mixture ratio. On the other hand, the single valve control produces a more marked effect on chamber pressure, due primarily to the change in total mass flow necessary to produce a change in mixture ratio when only the oxidizer of the fuel flow is controlled. Due to greater propellant density this effect is more pronounced for a single valve control in the oxidizer line. The curves of flow rates in the various lines of the system clearly show the large bypass flow rates associated with single valve control where the reduced gain results in the necessity for greater valve movement for corrective action. It should be noted that the bypass flows vary linearly with valve position as might be expected from the minor variation in the pressure drop across the valves. Also of some interest is the variation in the uncontrolled flow where a single control valve is used. Here the chamber pressure shift caused primarily by the variation of flow in the controlled line results in a slight change in injection overpressure for the uncontrolled line and this a small change in flow. This variation is always in such a direction as to aid the desired mixture ratio shift but the effect is quite small. Using the analog simulator data for the dual valve control system, the variation in engine thrust, F, from the design value of 150,000

pounds at r = 1.37, was calculated for the operating range of mixture ratios.

$$F = I_{sp}\dot{m}g$$

In the range, $1.25 < r < 1.47$, the variation was negligible. The maximum variation was found to be 0.7% at r = 1.5.

As previously mentioned the object of the final phase was to select controller parameters that would produce optimum system response commensurate with the desired requirements. The requirement limits are:

1. The control system be relatively insensitive to input frequencies greater than 50 radians/ second.

2. The maximum response rate to a step error signal of amplitude A, should not exceed A/10 over a period of .004 seconds.

Examination of the system diagram shown in Figure 8 indicates that if the chamber and feed lines are considered as a constant and only proportional amplification of the error signal is used the overall system is second order where

$$\Delta r = \frac{K_\tau}{s(T_\tau s+1)+K_\tau} \Delta R$$

with a natural frequency of

$$\omega_{n\tau} = \sqrt{\frac{K_\tau}{T_\tau}} \qquad \omega_n = \frac{\omega_{n\tau}}{t_s}$$

and damping ratio of

$$\zeta = \frac{1}{2\sqrt{K_\tau T_\tau}}$$

Here the time constant T_τ is that of the valve actuator while the gain K_V is composed of both actuator gain and basic system gain.

In order to meet the first requirement it was desired to select a valve

actuator with a natural frequency of approximately 50 radians per second. This insures an output attenuation of -12 decibels per octave beyond this point.

To meet the second requirement it was considered desirable to limit the response rate such that the maximum rate would be no more than 10% of the amplitude of a step input over the stay time of the chamber, .004 seconds.

The final criterion applied to desired response was that of best possible settling time; that is, the time for any error to be reduced to within 5% of its initial value.

Computer solutions for system response to a step input are shown in Figure 13 and 14, for several values of K_v and T. Figure 15 gives the variations of the response parameters as a function of controller gain for the two assumed actuator time constants of .01 and .02 seconds. The most noticeable variation between the two conditions is the decrease in settling time associated with the smaller time constant. Use of a valve actuator time constant of .01 seconds allows the realization of a setting time of .068 seconds as compared with the optimum time of .108 seconds for the case of an actuator with T = .02 seconds. In each case the controller gain for the optimum response is approximately the same, $K_v = 250$. The natural ability of the control mechanism to filter out inputs with frequencies greater than 50 radians per second is less for the faster responding control. Figure 16 shows the closed loop frequency response of the system with control parameters adjusted for optimum response with an actuator time constant $\tau = .01$ seconds. The frequency response attenuation falls off at -12 decibels per octave from a break frequency of 70 radians per second rather than the desired value of 50

radians per second. This is correctable by using a lower controller gain at the expense of a greater settling time; use of a valve actuator with a larger time constant, again at the expense of settling time; or use of a network filter to operate on the error signal. This last method presupposed complete actuator response up to 50 radians per second. For both values of the actuator time constant rate limitations were met when the settling time was a minimum.

Observation of the system response for initial conditions other than those at design mixture ratio of 1.37 revealed little change in the response characteristics. The only noticeable change was a small increase in damping due to the slight drop in the basic system gain associated with increasing valve actuator position.

The possible use of a dual bypass control system to effect stabilization of low frequency chamber pressure oscillations as well as performing as a propellant utilization control may be inferred from the study by Marble and Cox, Reference 10, where the use of a variable feed line capacitance was proposed. Bypass control is analogous to such a system if one considers that

$$\dot{m} - \dot{m}_i = \dot{m}_b = \frac{dC}{dt}$$

To be effective such a control would require good valve actuation response up to frequencies of 80 cycles per second, which calls for actuator time constants in the region of T = .0015 seconds. This is one order of magnitude beyond the assumptions made in this study.

Of considerable practical interest are certain "fail-safe" features of the proposed control system. In the event of a valve or actuator malfunction the maximum resulting change in propellant flow will depend upon the area of the

control valve orifive. For the application of such a system to the rocket vehicle of Appendix I the orifice areas required for operation at ideal conditions were:

$$\overline{A}_f = .266 \text{ in.}^2, \qquad \overline{A}_o = .303 \text{ in.}^2$$

These values correspond to valve positions of $\overline{\theta}$ and vary directly as $\overline{\theta}$. Examination of Figures 9, 10, and 11 indicates the feasibility of limiting the result of complete valve failure in the full open position by the choice of the available orifice area. Limitations on results of a complete closure may be obtained through proper choice of pump characteristics. Should only a single valve malfunction the control system will still be capable of limited corrective action through the use of the remaining valve. Provided that there has not been a complete failure, the remaining valve may well be able to both compensate for the effects of the failed valve and also effect a limited degree of propellant utilization.

CONCLUSIONS

From the preliminary investigation it is apparent that a propellant utilization scheme based on varying the operating mixture ratio is feasible. The allowable range over which the mixture ratio may be varied is a function of the type of propellant combination used in the engine.

Based on linearized simulation study of the proposed system as applied to a typical large rocket using a LOX-ETHYLAC propellant combination, the following conclusions are made:

1. Where the controller response was specified to avoid appreciable phase lags between line and chamber flows due to the combustion

time delay, the dynamics of the combined engine and propellant utilization system were determined, in the main, by the response characteristics of the controller.

2. Within the accuracy which the loading ratio is known the system is capable of producing empty burn out.

3. The "best response" design requires a controller gain of $K_v = 250$, with a time constant of $T = .01$ seconds. This response exhibited no overshoot and a settling time of .068 seconds.

4. Dual valve control is preferable to single valve control due to the fact it allows a larger range over which the mixture ratio can be varied without exceeding chamber pressure limitations or appreciably effecting thrust performance.

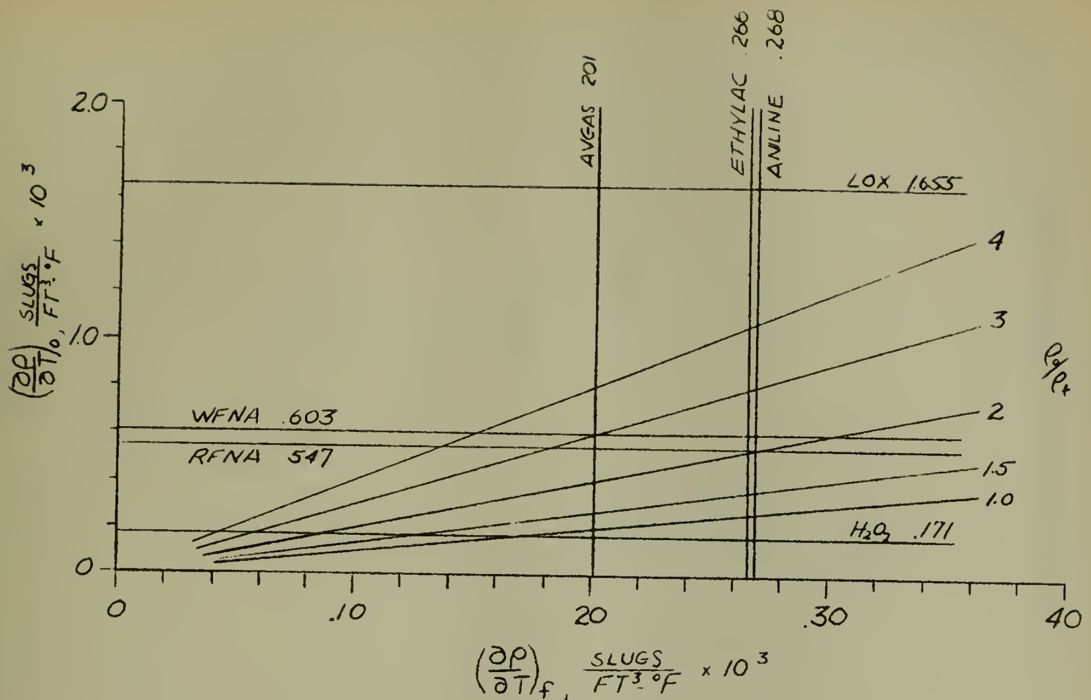
5. Engine performance in terms of thrust remains essentially unchanged. The maximum variation in thrust performance over the proposed range of mixture ratio control was 0.7%. In general the actual variation was negligible except at the extremes of the allowable range.

6. The system operation does not result in undesirable variations in chamber pressure. The maximum variation in chamber pressure over the range of mixture ratio control was 1% of the design value.

7. With a chamber pressure feedback in the control loop for dual valve control the allowable range of mixture ratio excursions may be extended.

8. If controller response times in the order of .002 seconds can be made practicable it appears the bypass control system for propellant utilization could serve the additional function of controlling low frequency combustion instability. An immediate consequence of this would be a desirable reduction in injection over-pressure.

9. The proposed system of dual valve control has a reliability advantage over single valve control is that corrective action may still be accomplished despite the malfunction of a single valve.



PROPELLANT COMBINATION	ρ_o/ρ_f
Lox and Avgas	1.5
Lox and Ethylac	1.3
H_2O_2 and Avgas	1.82
RFNA and Aniline	1.56
WFNA and Aniline	1.49

FIGURE 1 Temperature gradient ratios for several typical liquid propellant combinations.

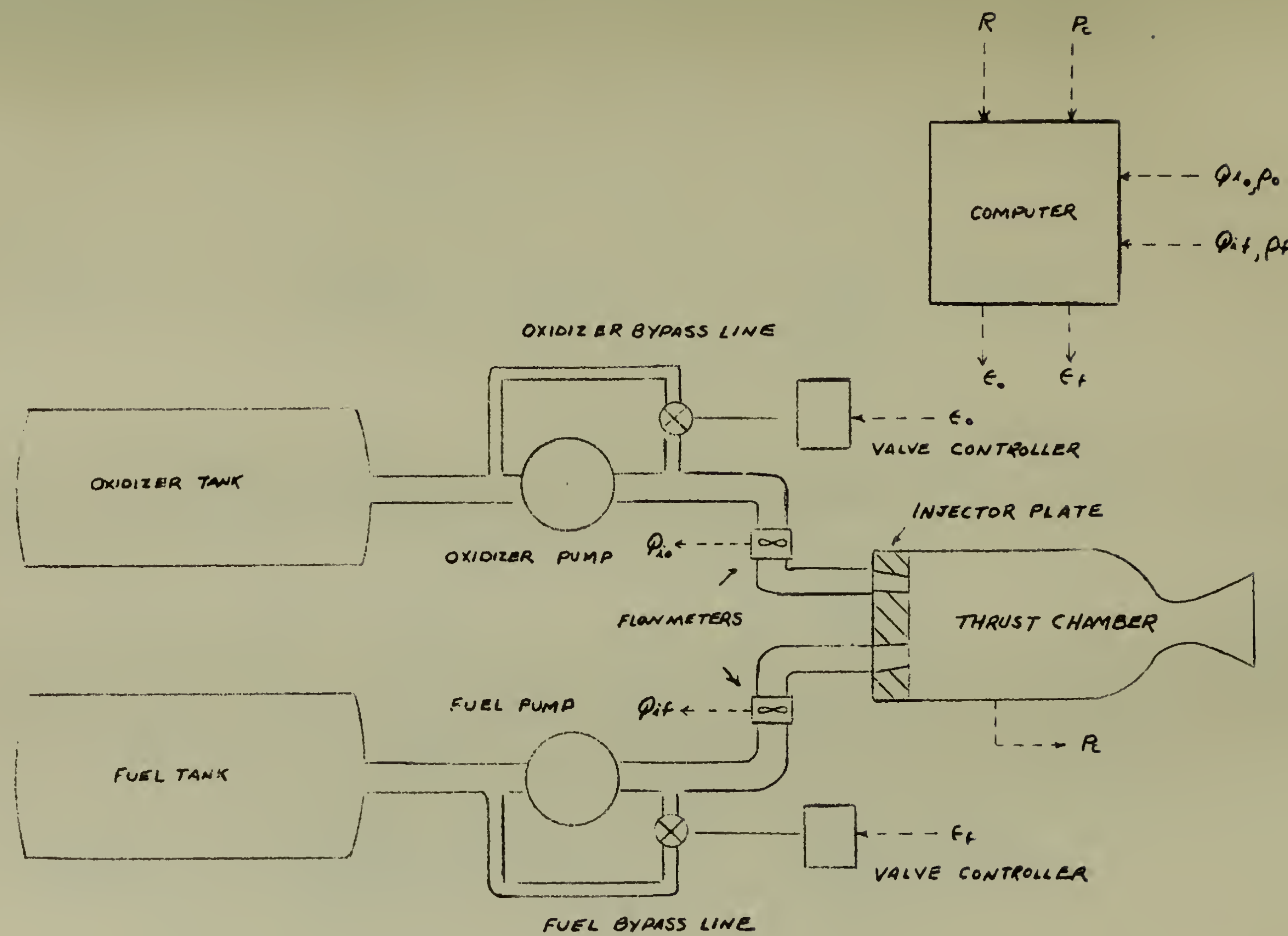


FIGURE 2 Schematic diagram of bipropellant rocket engine with bypass flow controls.

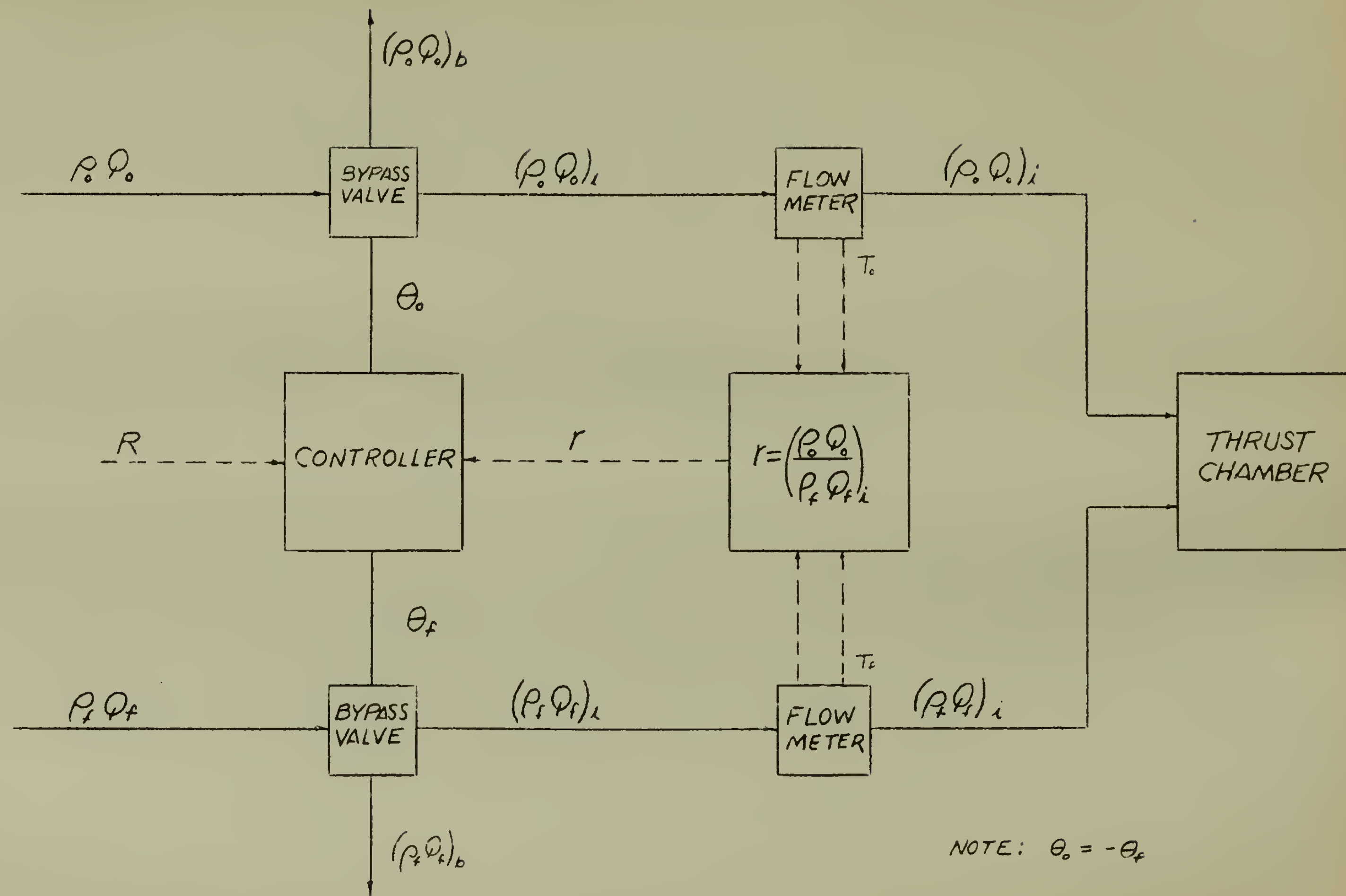


FIGURE 3 Schematic diagram for bypass flow control system.

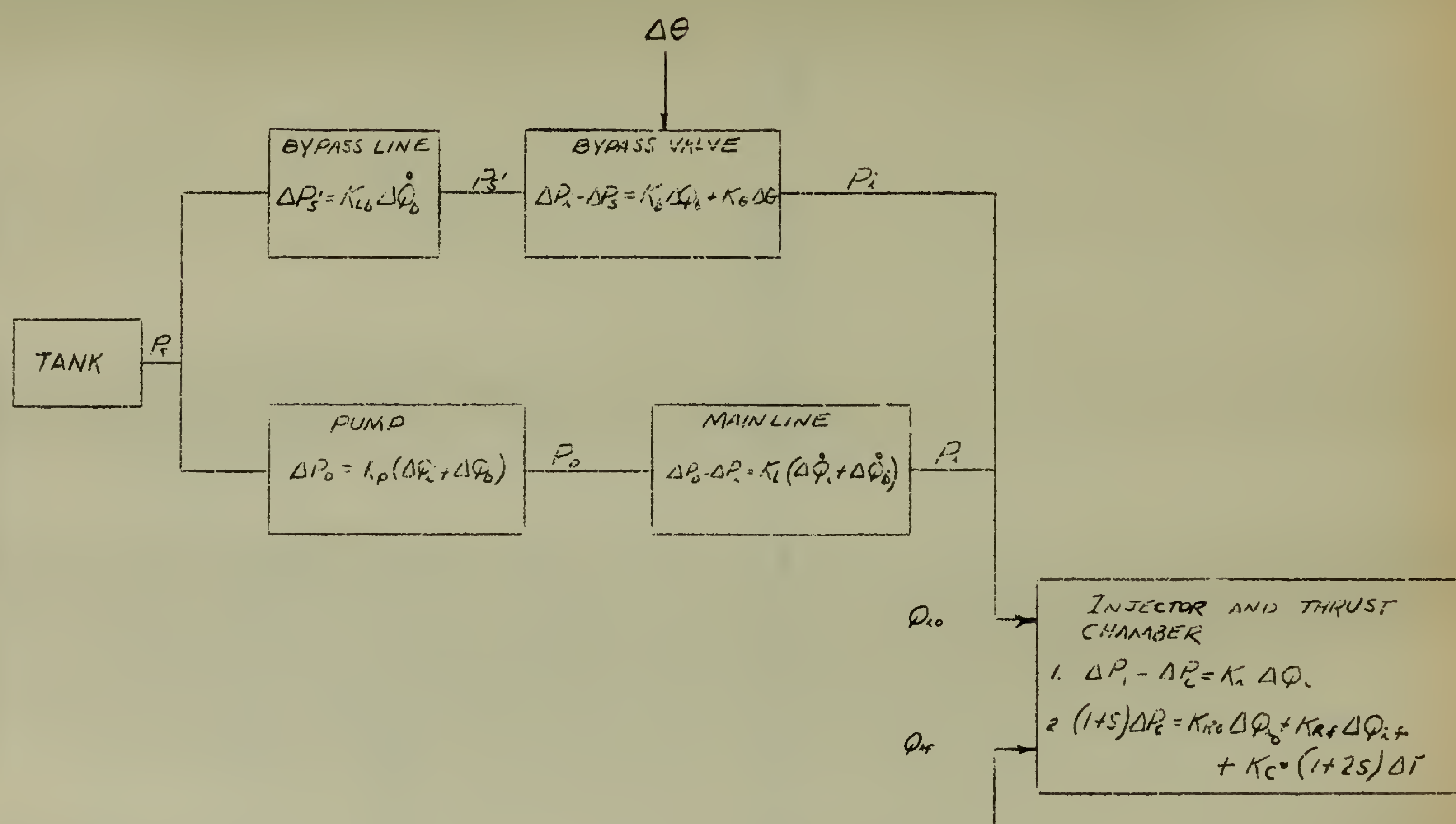


FIGURE 4 System equations for propellant utilization control.

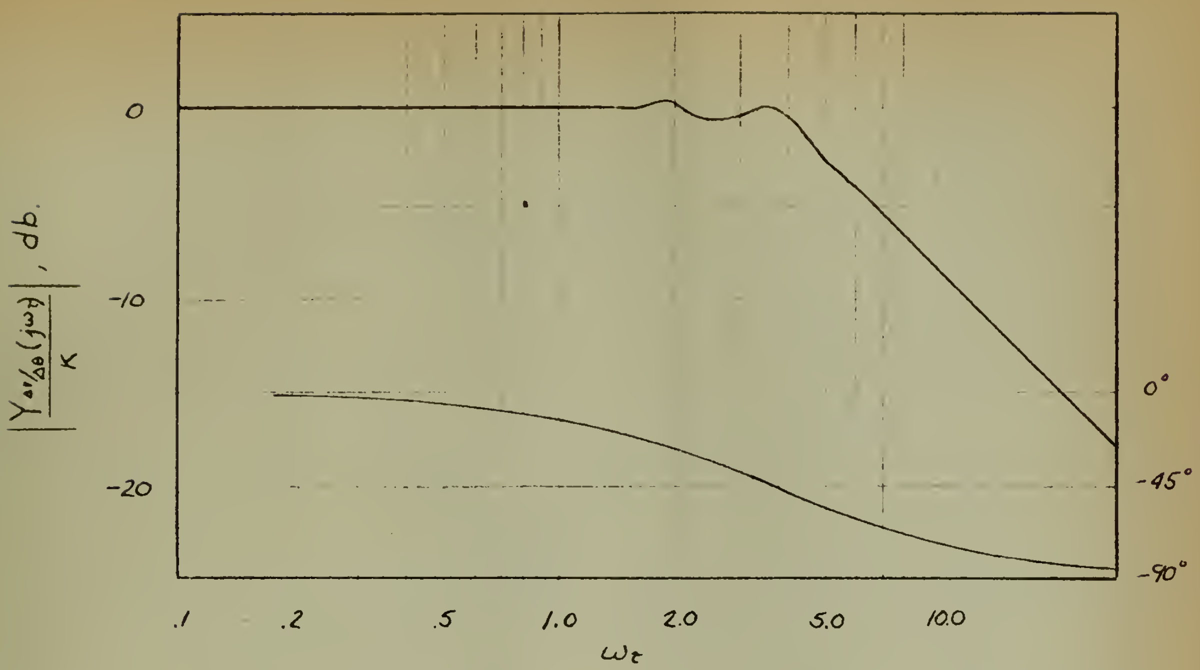


FIGURE 5 Predicted frequency response of mixture ratio to bypass valve position, open loop

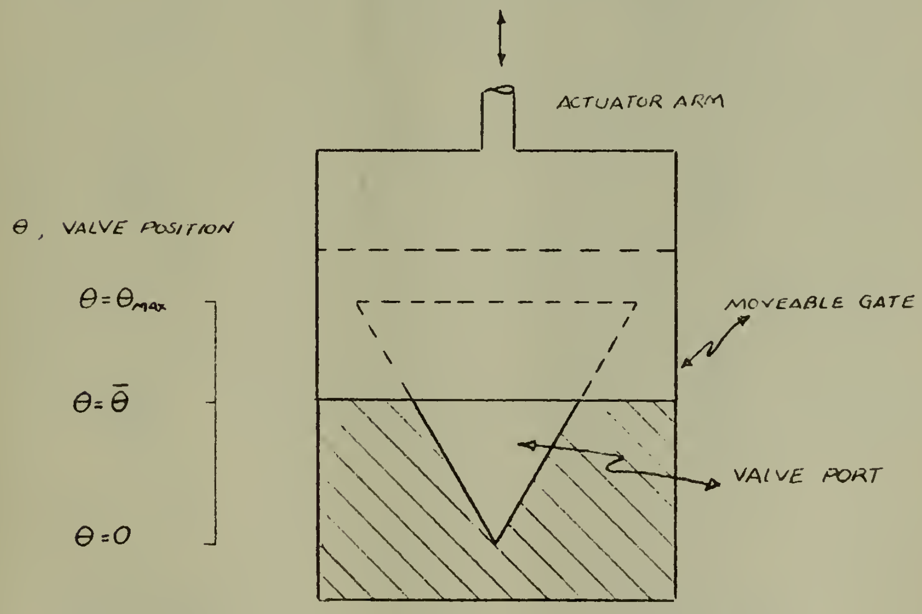


FIGURE 6 Schematic diagram of bypass valve

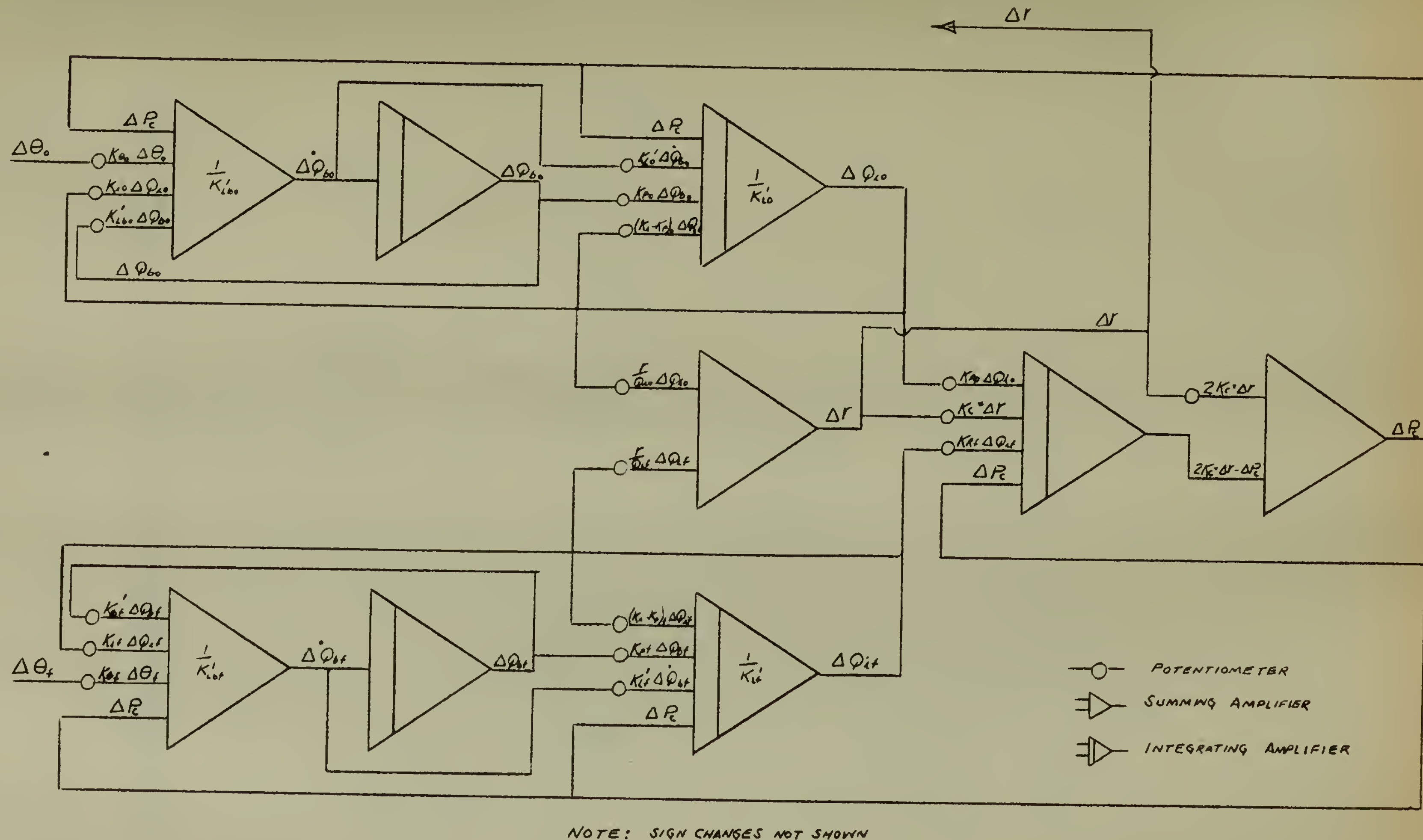


FIGURE 7 Analog simulator diagram for propellant utilization system without controller dynamics

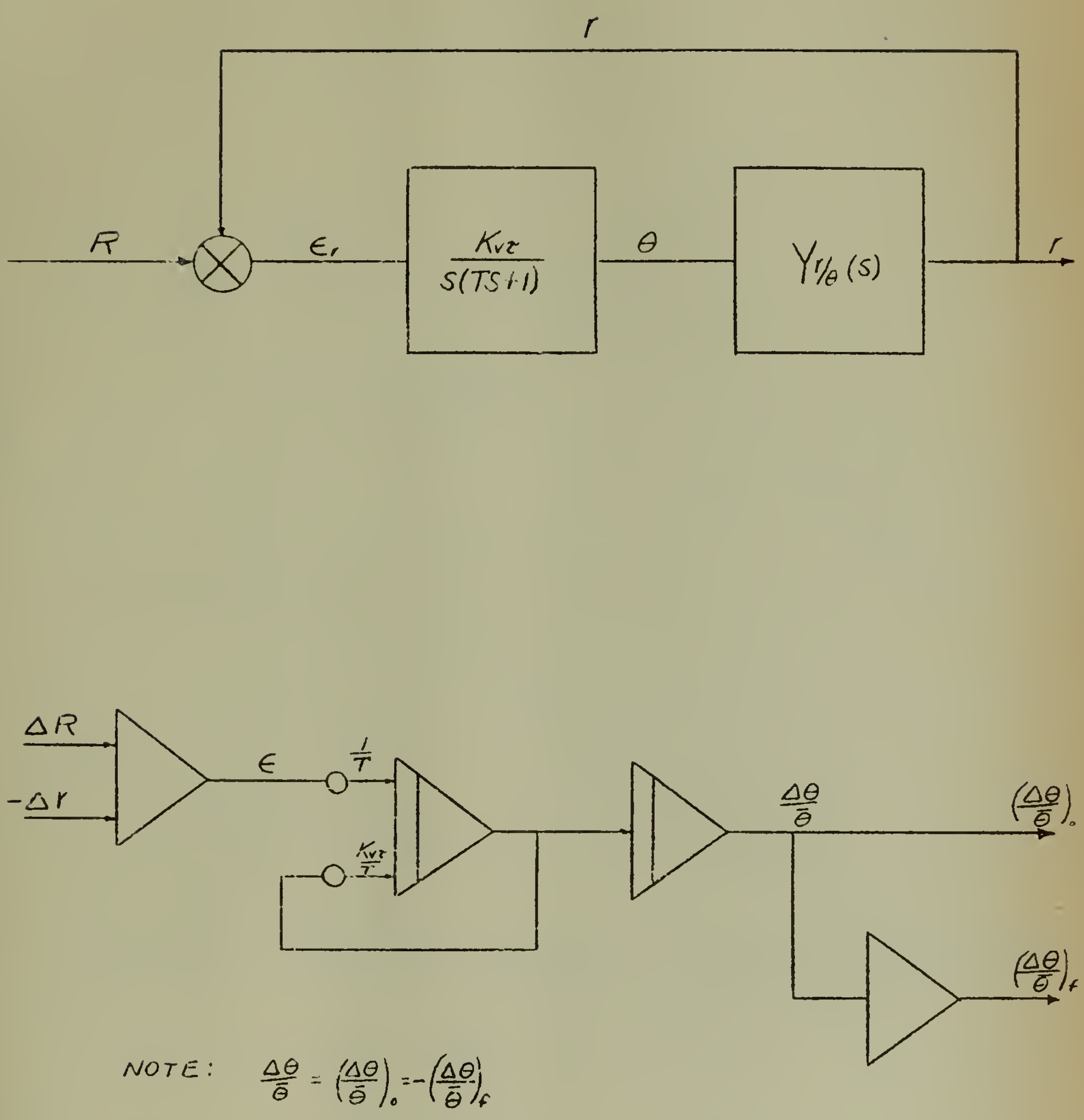


FIGURE 8 **Block diagram and simulator diagram for bypass valve control**

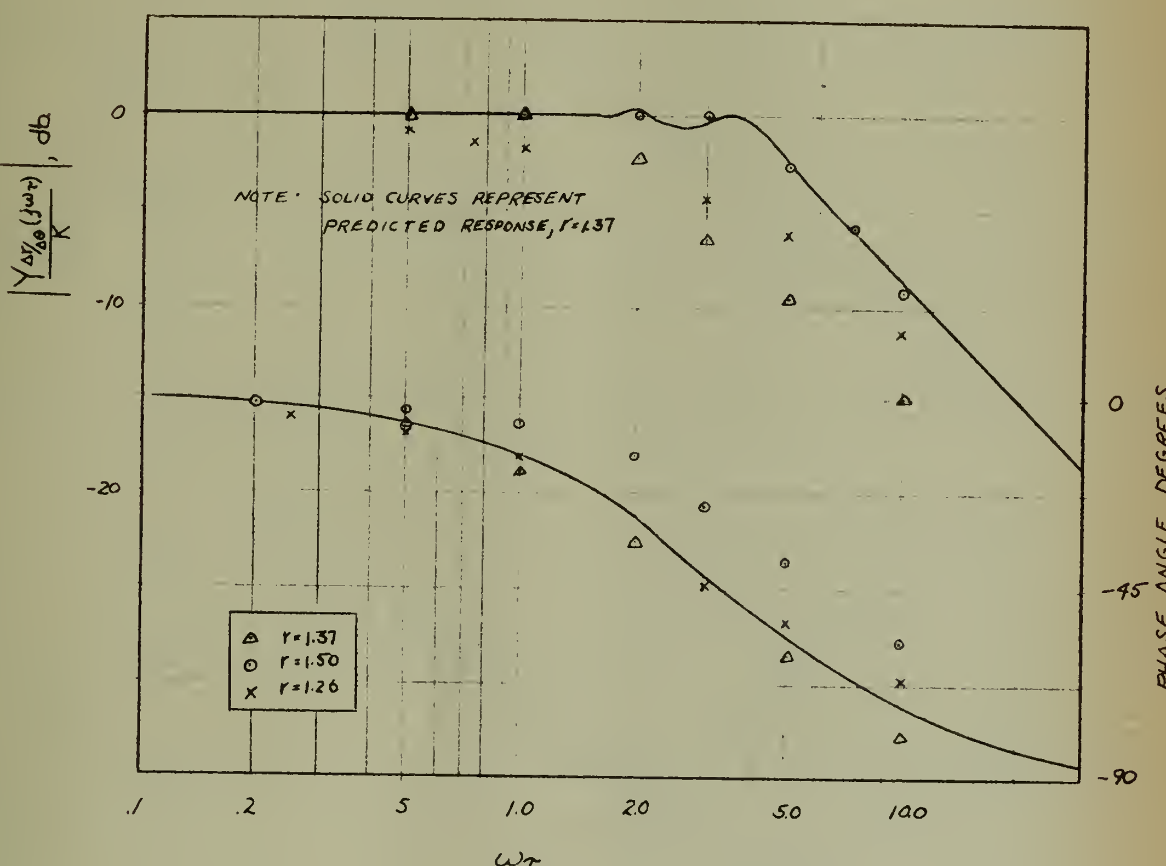


FIGURE 9 Actual frequency response of mixture ratio to bypass valve position, open loop

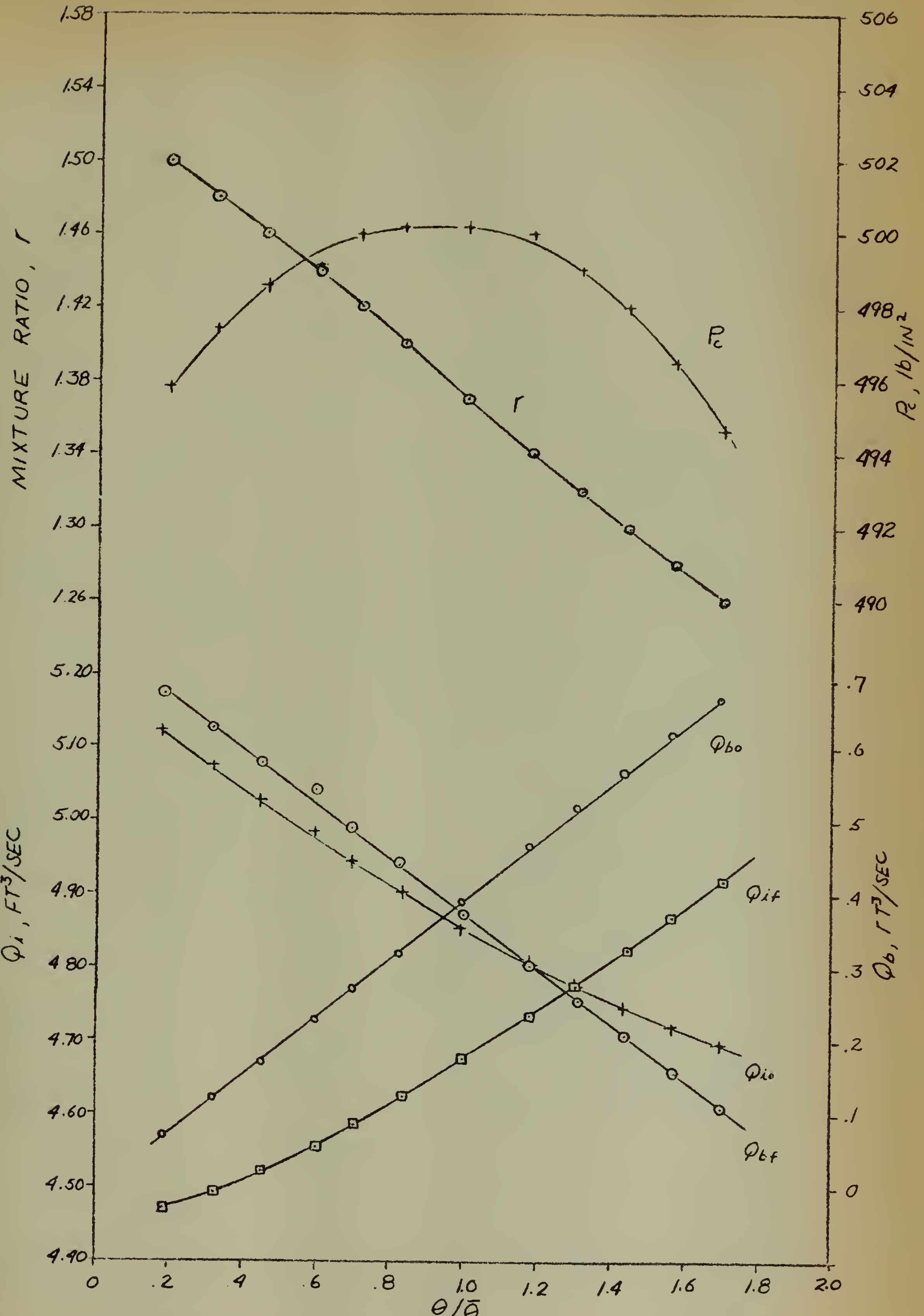


FIGURE 10 Steady state system operation for dual valve control

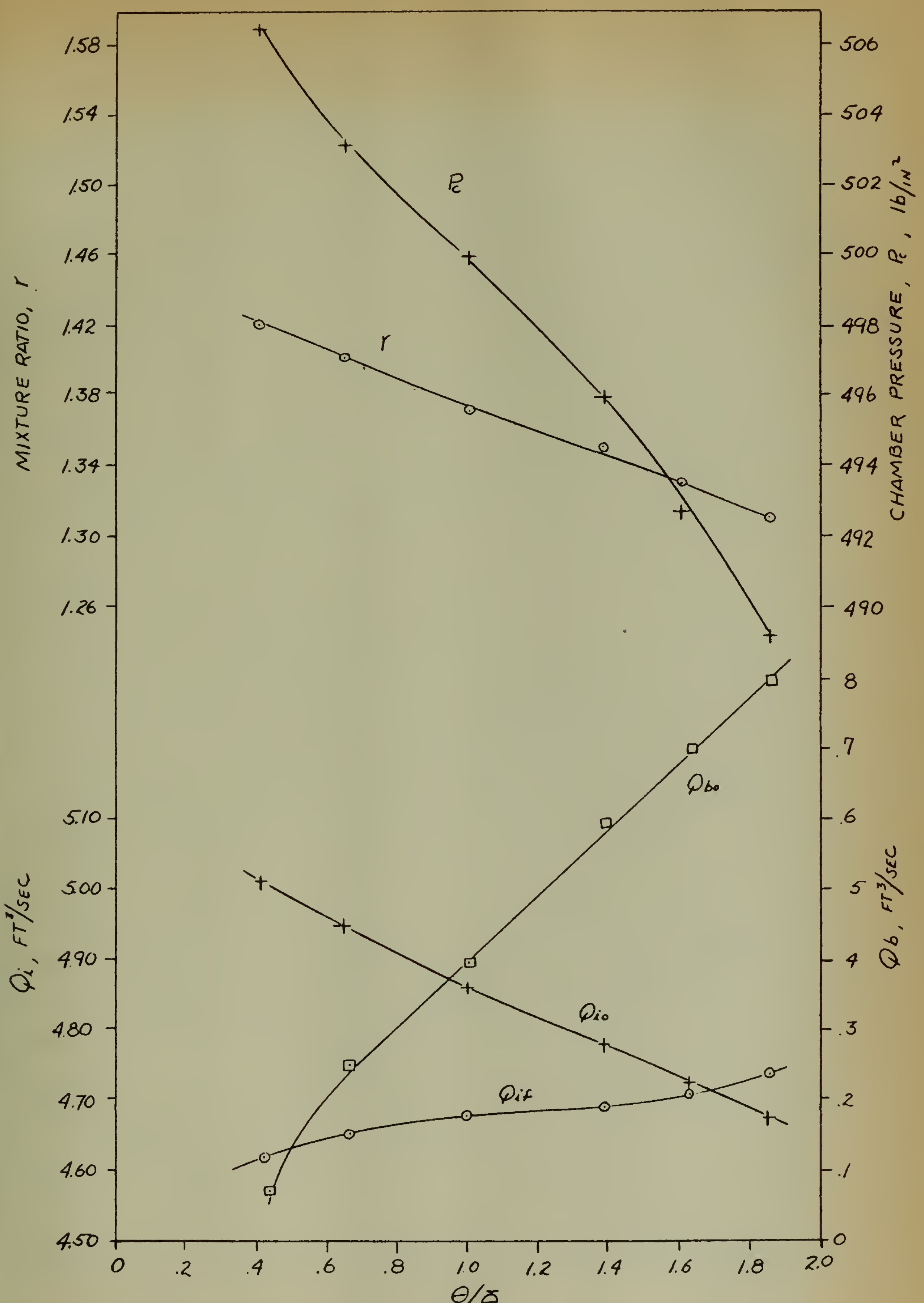


FIGURE 11 Steady state system operation for bypass control in oxidizer line only

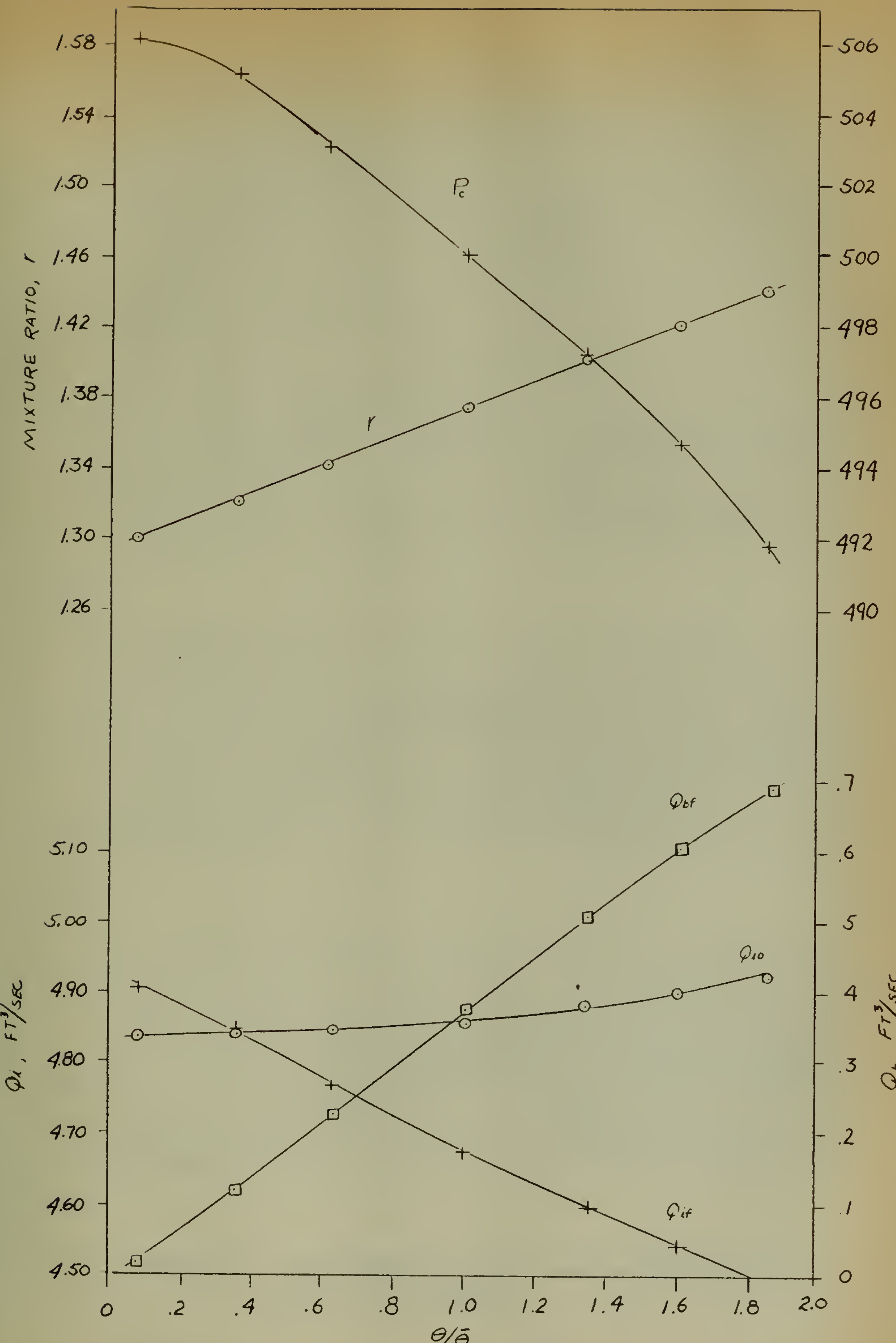


FIGURE 12 Steady state system operation for bypass control in fuel line only

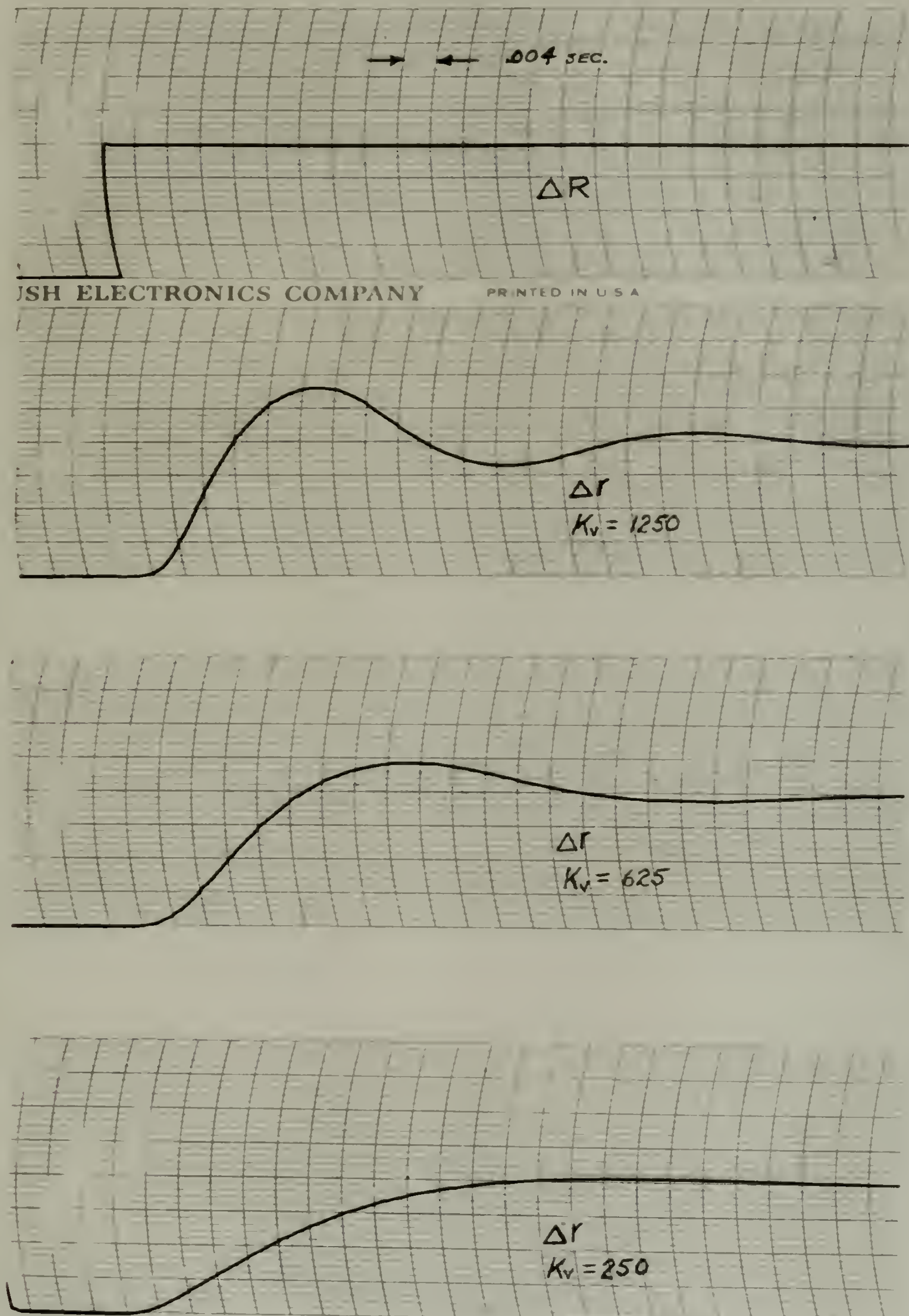


FIGURE 13 **Step response of mixture ratio for different values of gain; τ = .01 seconds**

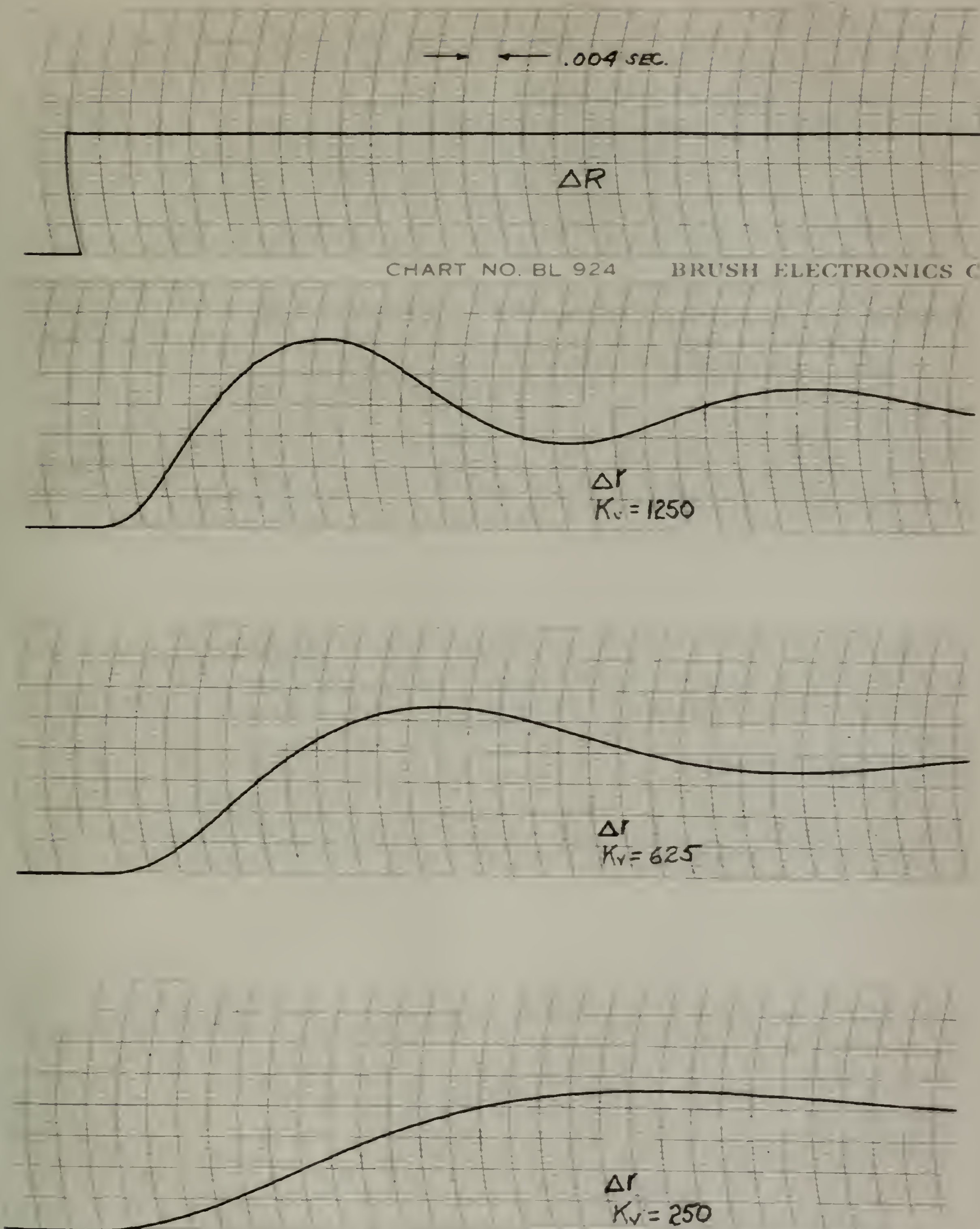
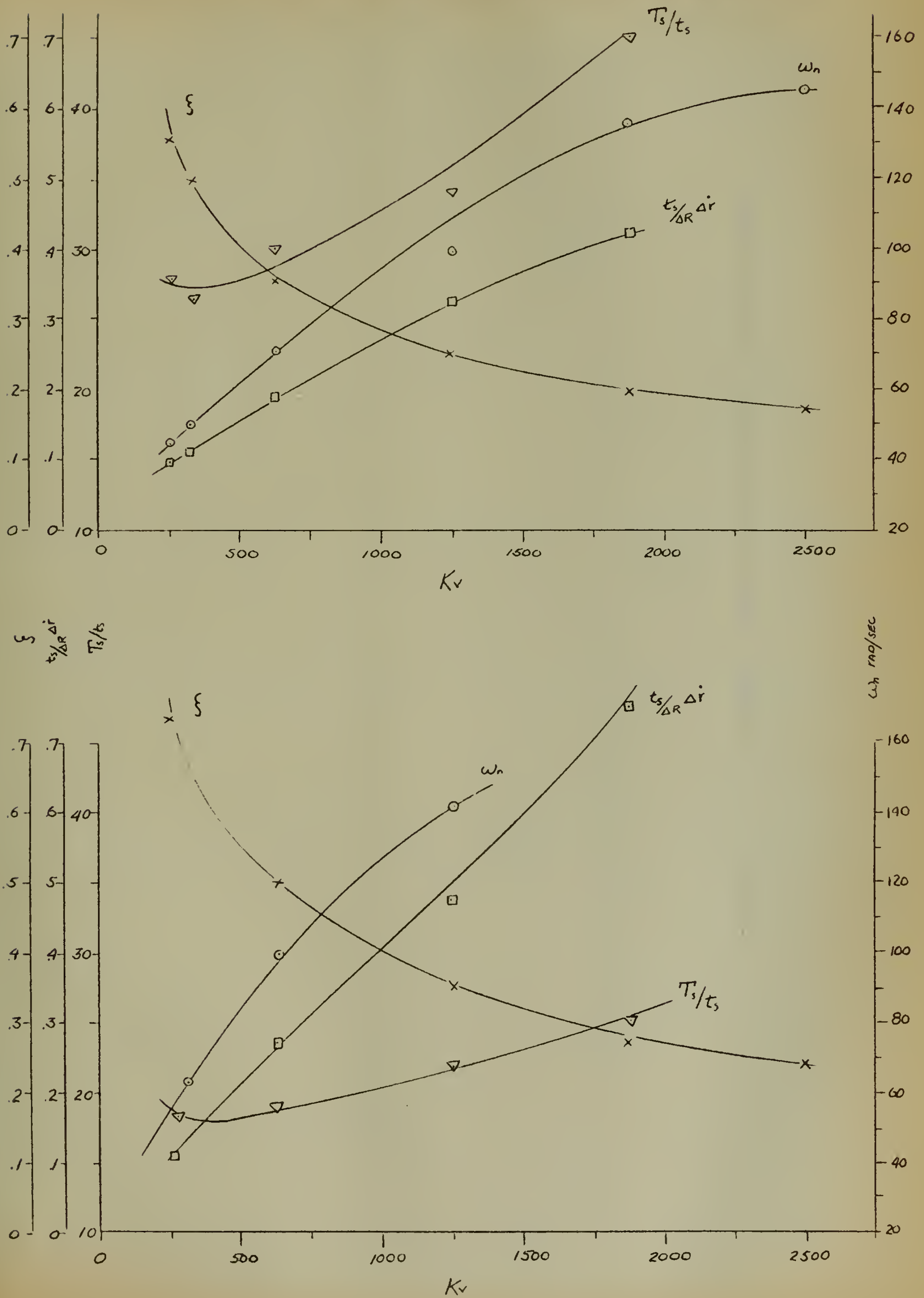


FIGURE 14 **Step response of mixture ratio for different values of gain; τ = .02 seconds**

FIGURE 15   Effect of controller gain on response parameters



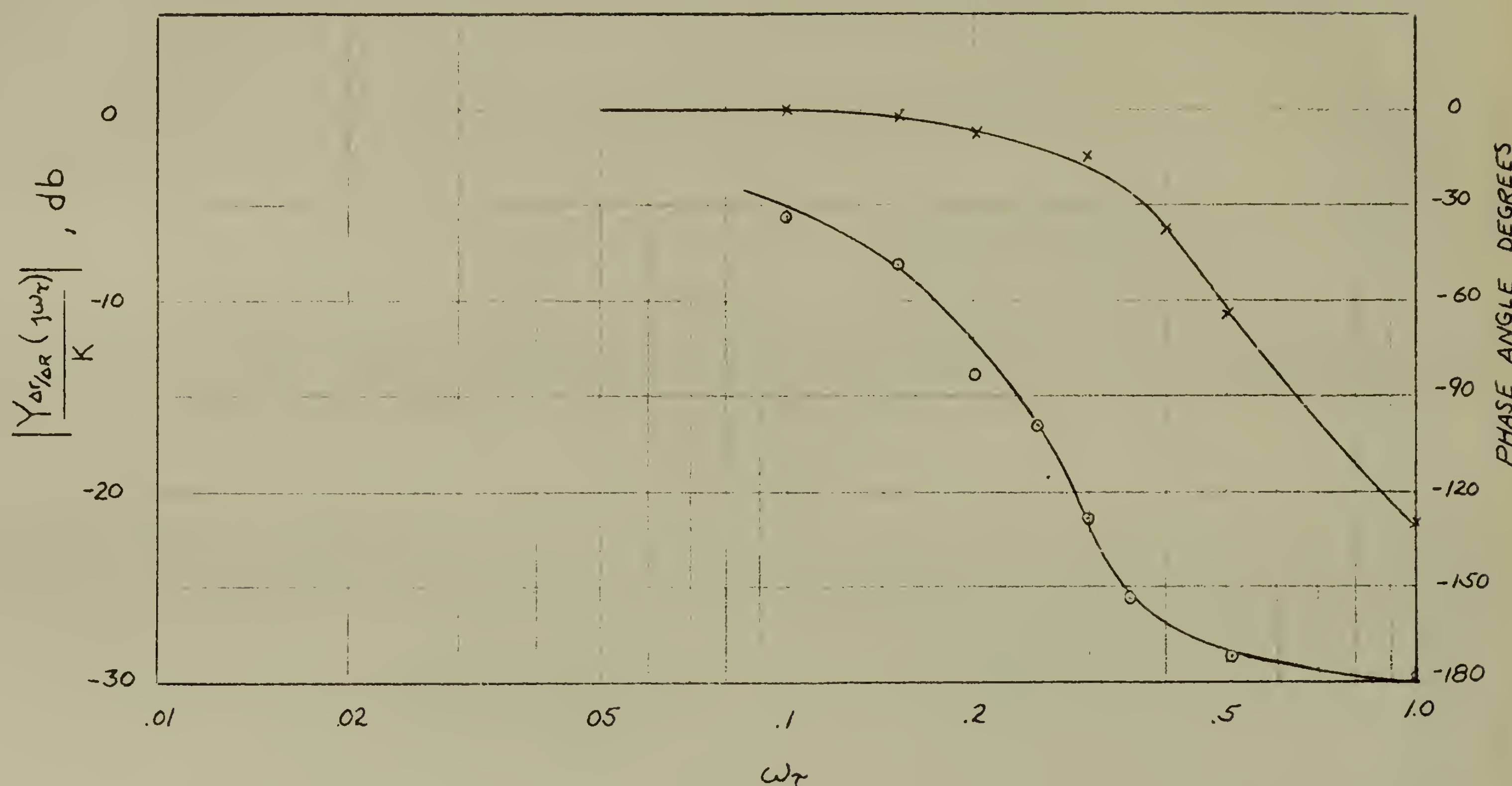


FIGURE 16 Frequency response of mixture ratio for entire system

ACKNOWLEDGEMENT

The authors wish to acknowledge their indebtedness for the council and assistance of Professor D. E. Rogers, of the University of Michigan in the preparation and presentation of this study.

REFERENCES

1. Gore, Marvin R. and Carroll, John J., "Dynamics of a Variable Thrust, Pump Fed, Bipropellant, Liquid Rocket Engine System," Journal of the American Rocket Society, vol. 27, January 1957, pp. 35-43.

2. Smith, B. N., "Perturbation Analysis of Low-Frequency Rocket Engine System Dynamics on an Analog Computer," Journal of the American Rocket Society, vol. 26, 1956, page 40.

3. Sanders, John C., Novik, David, and Hart, Clint E., "Effect of Dynamic Characteristics of Rocket Components on Rocket Control," Institute of the Aeronautical Sciences, Preprint No. 710, 1957.

4. Crocco, L., "Combustion Stability, Part I." Journal of the American Rocket Society, vol. 21, Nov. 1951, page 163.

5. Brousseau, J. A. Jr., "The Optimum Ratio of Propellants for a Liquid Bipropellant Rocket Operating Within a Mixture Ratio Tolerance." Journal of the American Rocket Society, vol. 26, 1956, page 106.

6. Reichel, R. H., "The Importance of Mixture Ratio Control for Large Rocket Vehicles," Journal of the American Rocket Society, vol. 25, 1955, page 291.

7. Barrere, M., Moutet, A., "Low-Frequency Combustion Instability in Bipropellant Rocket Motors - Experimental Study," Journal of American Rocket Society, vol. 26, 1956, page 9.

8. Rosen, M. W., Snodgrass, R. B., "Margin for Error" - "Space Flight Problems," Swiss Astronautical Society, 1953, page 60.

9. Crocco, L., Grey J., and Mathews, G. B., "Measurements of the Combustion Time Lag in a Liquid Bipropellant Rocket Motor," Journal of the American Rocket Society, vol. 26, 1956, page 20.

10. Marble, F. E., Cox, D. W. Jr., "Servo Stabilization of Low Frequency Oscillations in a Liquid Bipropellant Rocket Motor," Journal of the American Rocket Society, vol. 23, page 63.

11. Lee, Y. L., Gore, M. R., Ross C. C., "Stability and Control of Liquid Propellant Rocket Systems," Journal of the American Rocket Society, vol. 23, 1953, page 75.

12. Summerfield, M., "A Theory of Unstable Combustion in Luquid Propellant Rocket Motors," Journal of the American Rocket Society, vol. 21, September 1951, pp. 108-114.

13. Grey, J., Liu, F. F., "Methods of Flow Measurement," Journal of the American Rocket Society, vol. 23, 1953, page 133.

14. Grey, J., "Transient Response of the Turbine Flow Meter," Journal of the American Rocket Society, vol. 26, 1956, page 98.

15. Gebhardt, G. T., "What's Available for Measuring Mass Flow," Control Engineering, February 1957, page 90.

16. Sutton, George P., "Rocket Propulsion Elements," 1949, 2nd Ed., John Wiley and Sons, Inc., New York.

APPENDIX I.

NOMINAL DESIGN VALUES FOR A TYPICAL LARGE ROCKET VEHICLE

Thrust, lb.	150, 000
Take-off thrust-weight ratio	1. 5
Empty weight, lb.	20, 000
Take-off weight, lb.	100, 000
Propellants, Liquid oxygen-ethyl alcohol (with 25% H_2O)	
Flow rates at optimum mixture ratio, lb. /sec.	
Main oxidizer line	375
Oxidizer bypass line	28
Main fuel line	274
Fuel bypass line	21
Cross-section areas, $ft.^2$	
Main oxidizer line	. 2430
Oxidizer bypass line	. 0195
Main fuel line	. 2340
Fuel bypass line	. 0190
Line lengths, ft.	
Pump to injector	4
Bypass line	6
Fluid velocities in propellant lines, ft. /sec.	20
Alcohol density, $slugs/ft.^3$	1. 675
Liquid oxygen density, $slugs/ft.^3$	2. 210

Injection overpressure, psi	100
Thrust chamber parameters	
Characteristic velocity, ft./sec.	5,600
Characteristic length, ft.	8
Chamber pressure, psi	500
Optimum mixture ratio	1.37
Specific impulse, sec.	250
Chamber stay time, sec.	.004
Throat area, $in.^2$	209
Chamber volume, $ft.^3$	11.6

APPENDIX II

EFFECT OF PROPELLANT UTILIZATION ON PERFORMANCE

In many cases, depending on the application, it is desirable for a rocket to arrive at the burn out point in a minimum weight (mass) condition. For the bipropellant liquid rocket this optimum condition is achieved if both propellant components are exhausted at the same instant. Any deviation from this optimum will manifest itself in the form of increased burn out weight due to the mass of the excess component on board where the penalty of the increased burn out mass is a loss in performance.

Ideally, if perfect control were maintained over the launch loading ratio, R, (where R is the ratio of the initial oxidizer weight to the initial fuel weight) and the burning mixture ratio, r, an empty burn out should be achieved. If the initial weight of the propellant is W_p then

$$W_p = W_{o_i} + W_{f_i} = W_{o_i}\left(1 + \frac{1}{R}\right) = W_{f_i}(1+R)$$

and the burning rate of the propellant is

$$\dot{w}_p = \dot{w}_o + \dot{w}_f = \dot{w}_o\left(1+\frac{1}{r}\right) = \dot{w}_f(1+r)$$

If $\dot{W}_p$ is constant then the burning time t_b is

$$t_b = \frac{W_p}{\dot{w}_p} \text{ for } R = r, \quad t_b = \frac{W_p}{\dot{w}_p}\left(\frac{1+r}{1+R}\right) \text{ for } r < R, \quad t_b = \frac{W_p}{\dot{w}_p}\left(\frac{1+r}{1+R}\right)\frac{R}{r} \text{ for } r > R$$

Thus any time the condition R - r = O is not satisfied t_b will be changed.

For the case where r< R the fuel will be exhausted first and

$$t_b = t_{bf} = \frac{W_p}{\dot{w}_p}\left(\frac{1+r}{1+R}\right)$$

The result is that there will be residual oxidizer on board at burn out.

$$L_o = W_{o_i} - \int_0^{t_b} \dot{w}_p dt = \frac{W_p}{R+1}(R-r)$$

For the case where r R, the oxidizer will be exhausted first and there will be an excess of fuel on board at burn out.

$$t_b = t_{bo} = \frac{W_p}{\dot{w}_p}\left(\frac{1+r}{1+R}\right)\frac{R}{r}$$

$$L_f = W_{f_i} - \int_0^{t_b} \dot{w}_f dt = \frac{W_p}{R+1}\left(1 - \frac{R}{r}\right)$$

Figure II-1a shows the variation in L as a function of r/R for the assumed design given in Appendix I.

Considering an idealized vertical trajectory where drag has been neglected

$$\frac{W}{g}\frac{dV}{dt} = F - W$$

$$V_{(t)} = gI_{sp} \ln \frac{W_i}{W_i - \dot{w}_p t} - gt$$

$$dh = -gI_{sp} \ln \frac{W_i - \dot{w}_p t}{W_i} dt - gt\,dt$$

$$h_{(t)} = gI_{sp} \frac{W_i}{\dot{w}_p}\left\{\left(\frac{W_i - \dot{w}_p t}{W_i}\right)\left[\ln\left(\frac{W_i - \dot{w}_p t}{W_i}\right) - 1\right] + 1\right\} - \frac{gt^2}{2}$$

Substituting t_b for t in the above equations the velocity and altitude at burn out can be found as functions of r/R. Figures II-1 b and c shows the performance variations as a function of r/R for the assumed design given in Appendix I.

If some performance level is an absolute requirement it is then necessary to carry enough propellants to insure that neither fuel nor oxidizer will have been exhausted prior to reaching this desired performance level. This consideration may add much to the overall size of the rocket due to the high growth factor in rocket propulsion systems.

This is best illustrated by considering the velocity equation in the following form

$$V_{(t)} = g I_{sp} \ln \frac{W_i}{W_{(t)}} - g \frac{W_i - W_{(t)}}{\dot{W}_p}$$

For a given burn out velocity requirement it can be shown that

$$\frac{dW_i}{dW_b} = \frac{\frac{I_{sp}\dot{W}_p}{W_b} - 1}{\frac{I_{sp}\dot{W}_p}{W_i} - 1}$$

Where I_{sp} and are assumed to be constant. Exclusive of added tankage weight this factor is 13 for the design shown in Appendix I . As a general conclusion it is obvious from the above equation that for a lift-off acceleration of 1/2g this growth factor is in the order of three times the ratio of initial weight to burn out weight.

$$\frac{dW_i}{dW_b} \doteq 3\left(\frac{W_i}{W_b}\right)$$

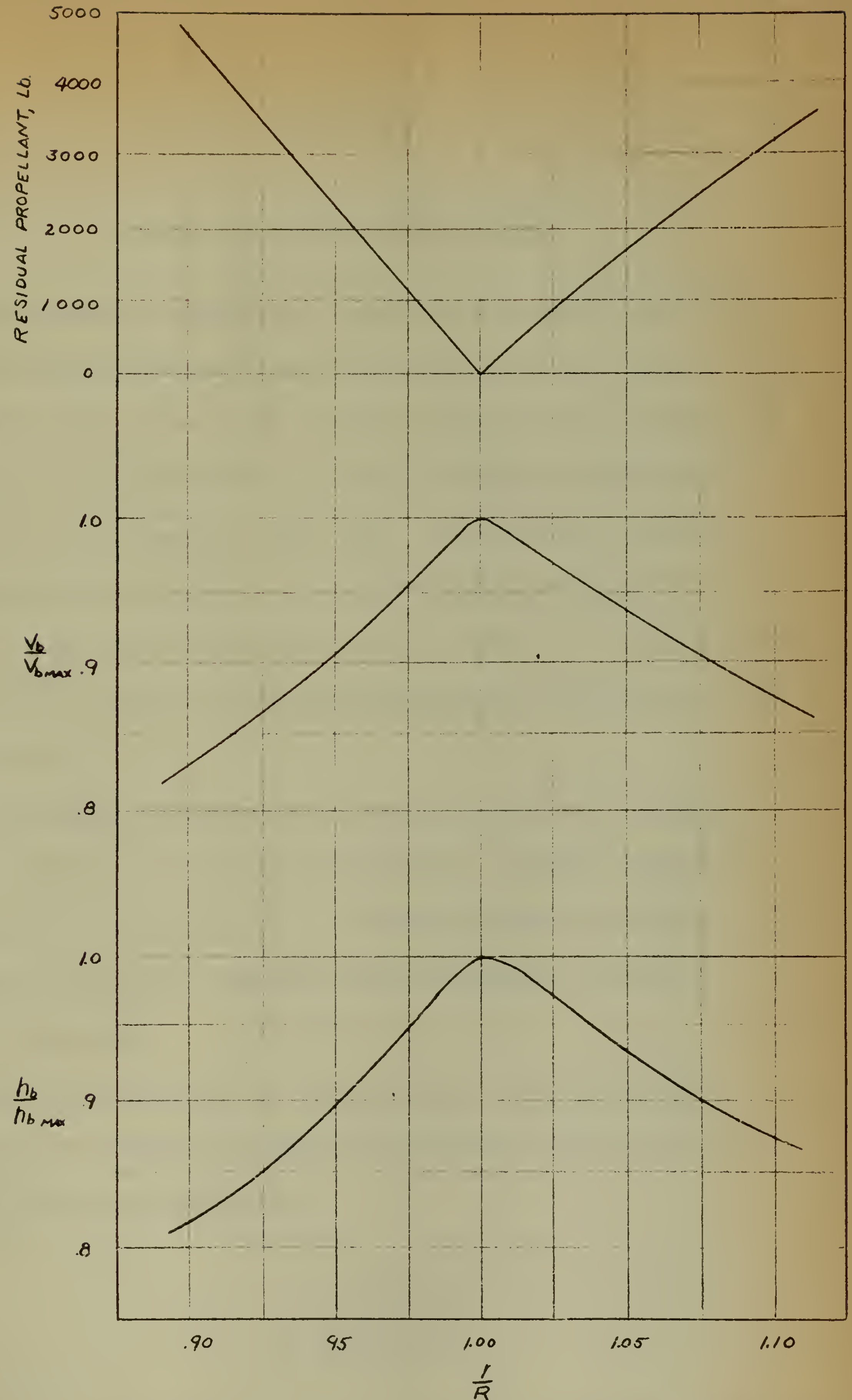


FIGURE II-1 Effect of improper propellant utilization on rocket performance at burn out

APPENDIX III

THRUST CHAMBER CONSIDERATIONS

Thermochemical equations may be used to obtain analytical expressions for the conditions existing within the thrust chamber relative to the performance of a rocket motor. Given some proposed combination of fuel and oxidizer and an assumed chamber pressure it is possible to obtain equilibrium values of $T, \mathcal{M}$, and γ as a function of mixture ratio. The actual values of the parameters will differ for different propellant combinations but in general are observed to follow the general pattern shown in Figure III-1. Specific examples of performance charts for several different propellant combinations are shown in Reference 16.

The results of the thermochemical analysis will not vary appreciably with the chamber pressure assumed providing the combustion equilibrium equations are not pressure sensitive. With the assumption of pressure insensitivity the chamber conditions can be considered as a function of mixture ratio alone.

One of the basic performance parameters of a rocket motor is the characteristic velocity c*, which can be expressed as a function of the combustion product gas properties as

$$C^* = \frac{1}{\gamma} \sqrt{\frac{\gamma g R' T/\mathcal{M}}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}$$

With T, $\mathfrak{m}$, and γ reduced to functions of mixture only it is seen that c* is also a function of mixture ratio. For many bipropellant systems γ may be considered constant over the range of r under consideration, thus

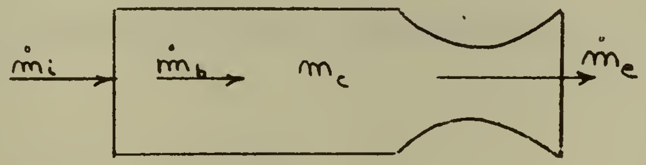
$$c^* = K_\gamma \sqrt{\frac{T}{\mathfrak{m}}}$$

A plot of $\sqrt{T/\mathfrak{m}}$ as a function of r typically results in a double-valued cure such as shown in Figure III-2. From the plot of $\sqrt{\frac{T}{\mathfrak{m}}}$ derived from thermochemical considerations it is possible to obtain an analytic expression for c* in terms of mixture ratio by the use of various curve fitting techniques.

Figure III-2 shows the variations of $\sqrt{\frac{T}{\mathfrak{m}}}$ with r for a liquid oxygen-ethyl alcohol (with 25% water) propellant combination. Here a parabolic approximation has been assumed to be valid over a 10% range of mixture ratio as measured from $\bar{r}$ = 1.37. The resulting values of c* over this band of mixture ratios are shown in Figure II-3. Based on the parabolic approximation for $\sqrt{\frac{T}{\mathfrak{m}}}$, c* is given as

$$\left(\frac{c^*}{\bar{c}^*} - 1\right) = -.26\left(\frac{r}{\bar{r}} - 1\right)^2$$

The dynamics of the combustion chamber are developed from the consideration of the requirements for mass flow conservation through the chamber.



Referring to the above sketch; under steady state conditions the flow through the chamber may be expressed as

$$\dot{m}_i(t-t_c) = \dot{m}_b(t) = \dot{m}_e(t) = \bar{\dot{m}}$$

where t_c represent the time delay between injection and combustion. For non-steady conditions the flow is expressed as

$$\dot{m}_i(t-t_c) = \dot{m}_b(t) = \frac{d}{dt} m_c(t) + \dot{m}_e(t)$$

If the time lag t_c is neglected and the flow at each point expressed as a perturbation about the steady state value the equation becomes

$$\bar{\dot{m}} + \Delta\dot{m}_i = \frac{d}{dt} m_c + \bar{\dot{m}} + \Delta\dot{m}_e$$

which can be rewritten as

$$\frac{\Delta\dot{m}_i}{\bar{\dot{m}}} = \frac{d}{dt}\frac{m_c}{\bar{\dot{m}}} + \frac{\Delta\dot{m}_e}{\bar{\dot{m}}}$$

Introducing a new time variable τ such that $\tau = t/t_s$, then

$$\frac{d}{dt} = \frac{d}{d\tau}\frac{d\tau}{dt} = \frac{1}{t_s}\frac{d}{d\tau}$$

and

$$\frac{\Delta\dot{m}_i}{\bar{\dot{m}}} = \frac{d/d\tau \; m_c}{t_s\,\bar{\dot{m}}} + \frac{\Delta\dot{m}_e}{\bar{\dot{m}}}$$

Here t_s is chosen as the stay time of the flow in the chamber. Therefore

$$\frac{\Delta\dot{m}_i}{\bar{\dot{m}}} = \frac{d}{d\tau}\frac{m_c}{\bar{m}_c} + \frac{\Delta\dot{m}_e}{\bar{\dot{m}}}$$

The fluid mass in the chamber may be represented as

$$m_c = \bar{m}_c + \Delta m_c$$

where

$$\frac{d}{d\tau} m_c = \frac{d}{d\tau} \Delta m_c$$

Thus

$$\frac{\Delta \dot{m}_i}{\bar{\dot{m}}} = \frac{d}{d\tau} \frac{\Delta m_c}{\bar{m}_c} + \frac{\Delta \dot{m}_e}{\bar{\dot{m}}}$$

The fluid mass in the chamber is given by

$$m_c = \frac{P_c V_c}{R' \frac{T}{m}}$$

and by substituting the previously determined expression for c*

$$m_c = \frac{K_r^2 V_c}{R'} \frac{P_c}{C^{*2}} = K_m \frac{P_c}{C^{*2}}$$

and

$$\frac{\Delta m_c}{\bar{m}_c} = \frac{\Delta P_c}{\bar{P}_c} - \frac{2 \Delta C^*}{\bar{C}^*}$$

For steady nozzle flow

$$\dot{m}_e = \frac{P_c A_t}{C^*}$$

and

$$\frac{\Delta \dot{m}_e}{\bar{\dot{m}}} = \frac{\Delta P_c}{\bar{P}_c} - \frac{\Delta C^*}{\bar{C}^*}$$

The flow through the chamber is now expressed in linear form for small perturbations about the steady state condition as

$$\frac{\Delta \dot{m}_i}{\bar{\dot{m}}} = \frac{d}{d\tau} \left(\frac{\Delta P_c}{\bar{P}_c} - \frac{2 \Delta C^*}{\bar{C}^*} \right) + \frac{\Delta P_c}{\bar{P}_c} - \frac{\Delta C^*}{\bar{C}^*}$$

Using the operator s to signify $d/d\tau$ the flow equation becomes

$$\frac{\Delta \dot{m}_i}{\bar{\dot{m}}} = (1+s) \frac{\Delta P_c}{\bar{P}_c} - (1+2s) \frac{\Delta C^*}{\bar{C}^*}$$

The injector flow may be considered in terms of the individual propellant flows and the mixture ratio from the following relationships.

$$r = \frac{\dot{m}_o}{\dot{m}_f}\,, \qquad \dot{m}_i = \dot{m}_o + \dot{m}_f$$

Thus the injector flow may be expressed in two forms

$$\dot{m}_i = \dot{m}_o\left(1 + \frac{1}{r}\right)$$

$$\dot{m}_i = \dot{m}_f\,(1 + r)$$

Therefore

$$\frac{\Delta \dot{m}_i}{\bar{\dot{m}}} = \frac{\Delta \dot{m}_o + \Delta \dot{m}_f}{\bar{\dot{m}}} = \frac{\Delta \dot{m}_o}{\left(1 + \frac{1}{r}\right)\bar{\dot{m}}_o} + \frac{\Delta \dot{m}_f}{(1 + r)\,\bar{\dot{m}}_f}$$

or in terms of volumetric flow where $\dot{m} = \rho Q$

$$\frac{\Delta \dot{m}_i}{\bar{\dot{m}}} = \frac{\Delta Q_{io}}{\left(\frac{r+1}{r}\right)\bar{Q}_{io}} + \frac{\Delta Q_{if}}{(1+r)\,\bar{Q}_{if}}$$

Substituting this express in the chamber flow equation gives

$$\frac{\Delta Q_{io}}{\left(\frac{r+1}{r}\right)\bar{Q}_{io}} + \frac{\Delta Q_{if}}{(r+1)\,\bar{Q}_{if}} = (1+S)\frac{\Delta P_c}{\bar{P}_c} - (1+2S)\frac{\Delta c^*}{\bar{c}^*}$$

If the variation in c* is assumed to be uniform throughout the chamber and, as before, the combustion lag time is neglected the last term on the right side of the above equation may be written in terms of injection mixture ratio

$$(1+2S)\frac{\Delta c^*}{\bar{c}^*} = \frac{1}{\bar{c}^*}\,\frac{dc^*}{dr}\,(1+2S)\,\Delta r$$

and in final form the chamber equation becomes

$$(1+S)\,\Delta P_c = K_{Ro}\,\Delta Q_{io} + K_{Rf}\,\Delta Q_{if} + K_{c^*}\,(1+2S)\,\Delta r$$

where

$$K_{Ro} = \frac{\bar{P}_c}{\bar{Q}_{io}}\left(\frac{r}{r+1}\right)$$

$$K_{Rf} = \frac{\bar{P}_c}{\bar{Q}_{if}}\left(\frac{1}{r+1}\right)$$

$$K_{c^*} = \frac{\bar{P}_c}{\bar{c}^*}\,\frac{dc^*}{dr}$$

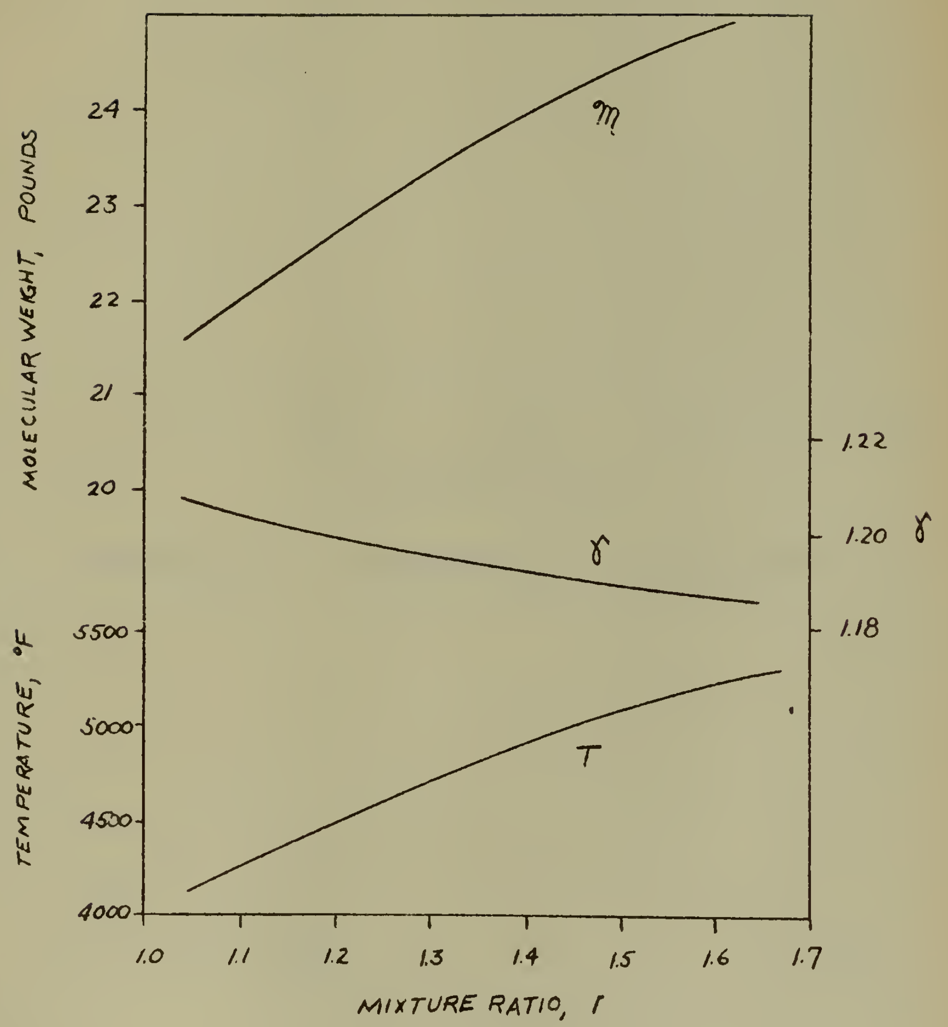


FIGURE III-1 Typical performance parameters as a function of mixture ratio for a liquid bipropellant rocket engine

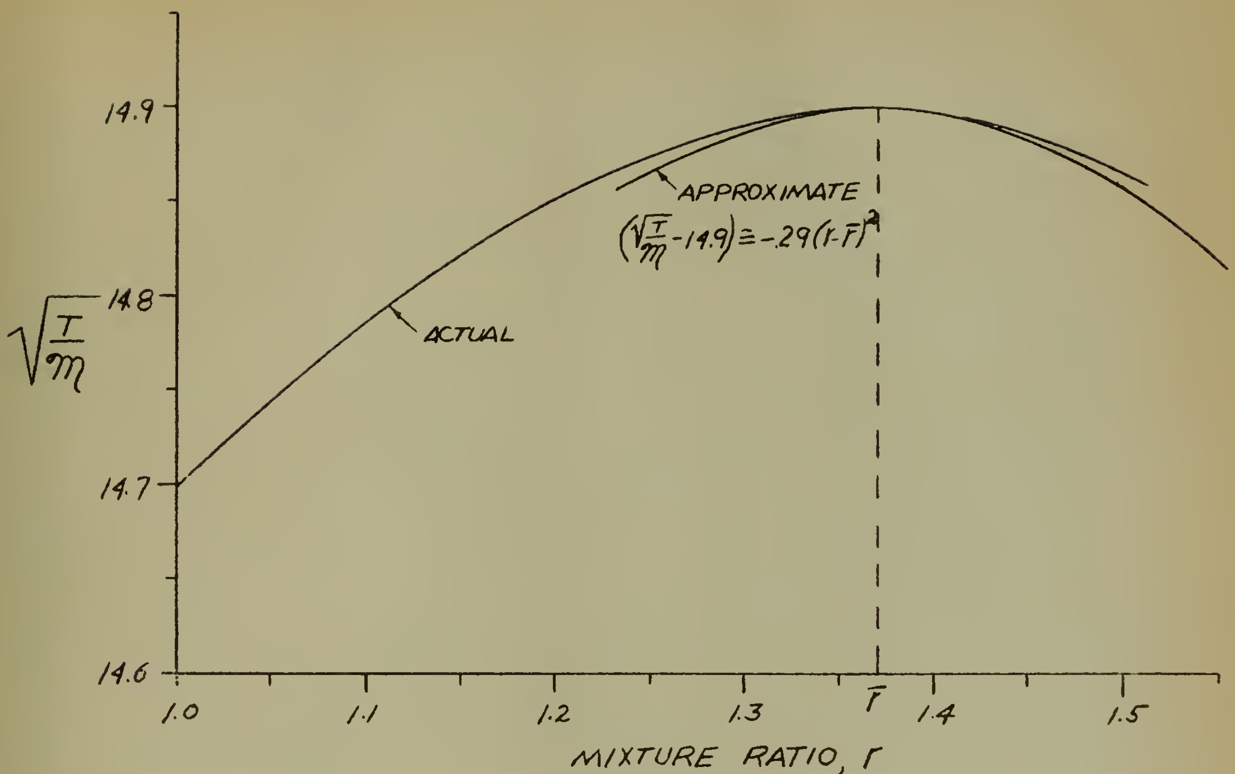


FIGURE III-2 Variation of $\sqrt{\frac{I}{m}}$ with mixture ratio

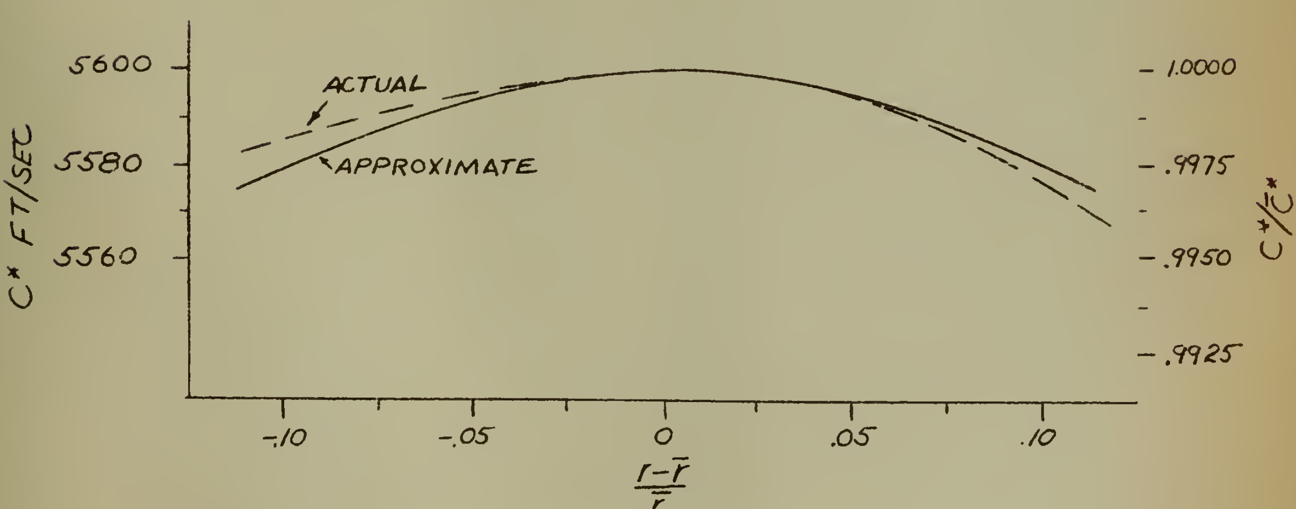


FIGURE III-3 Variation of characteristic velocity, c* with mixture ratio

NAVAL POSTGRADUATE SCHOOL
...3 5002

Thesis
K145
c.1
35888
Kastner
A proposal for the control of propellant utilization in a liquid bipropellant rocket engine.

DUDLEY KNOX LIBRARY
3 2768 00031360 5